On OSTBC Codes for LTE-A Systems-Design and Performance Evaluation

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Abstract

Long Term Evolution-Advanced (LTE-A) is a Fourth Generation (4G) standard of wireless communications that introduces high data rate, high performance, and low delay. These features of LTE-A resulted from the new techniques developed for wireless communications such as Multiple-Input Multiple-Output (MIMO) technique. At the heart of this technique is the Space Time Codes, which were developed by the researchers in recent decades to achieve the mentioned features. The designs of OSTBC codes for MIMO systems having any number of transmit antennas and any number of receive antennas have attracted the attention of many researchers. Based on the theory of real and complex orthogonal designs, this paper deals with the design of real and complex OSTBC codes to be used with real signal set constellation such PAM and complex signal constellation such as PSK and QAM. Real and complex OSTBC codes for MIMO systems with two, three, four, five, six, seven and eight transmit antennas and any number of receive antennas, are presented. Simple linear processing ML decoders are derived and presented. The used channel is Rayleigh fading channel MIMO and assumed to behave in a "quasi-static" fashion. Finally, the performances of OSTBC schemes were evaluated and compared in terms of the Bit Error Rate (BER) and Signal to Noise Ratio (SNR). The environment of simulation is MATLAB which is a powerful tool for mathematical calculation and system simulation. The methods of modulations chosen are QPSK, 16QAM, 64QAM, and 128QAM with gray scale mapping.
Keywords: LTE-A, MIMO, STBC, OSTBC, Rayleigh Fading, BER.

1. Introduction

In recent years, Space-Time coding (STC) technique considered one of the most important Multi Input Multi Output (MIMO) techniques [1-3]. STC can combat channel fading by transmitting several replicas of the same information through each antenna. By doing this,

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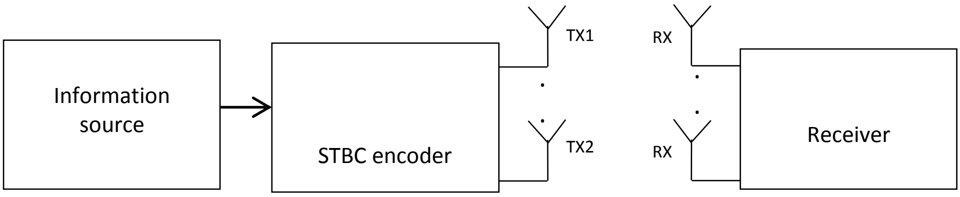
the probability of losing the information decreases exponentially [4]. The block diagram of a typical STBC encoding system is given in figure 1 [5].



Figure(1): Block diagram of typical STBC encoder

To transmit b bits, a modulation scheme is used that maps every b bits to one symbol from a constellation with 2^b symbols. The constellation can be any real or complex constellation, for example PAM, PSK, QAM, and so on. In what follows, the recent published literature related to the research topic is reviewed.

MIMO technologies use multiple antennas at both the transmitter and receiver to improve communication performance by offering significant increases in data throughput and link range without additional bandwidth or transmit power [5-7]. From the reviewed published literature, it is mentioned in [8] that the first bandwidth efficient transmit diversity scheme had been proposed in [9] where the authors make use of a special case of delay diversity proposed in [10]. In [11], space-time coding schemes has been proposed. In [12], first breakthrough, Alamouti had proposed a new space-time block coding technique utilizing a two-branch transmit diversity to code and transmit the data over two independent channels. Furthermore, it was shown that either using the simple Alamouti decoder [13] or the Maximum Likelihood (ML) [14] decoder. The received copies of the noisy signals can be easily combined and decoded. The decoding complexity of the Alamouti decoder has been demonstrated to be linear. The transmission scheme proposed by Alamouti (2×1 transmit diversity) was later generalized in [11, 15, 16] so that an arbitrary number of transmitting antennas can be employed and yet full diversity can be achieved, it is assumed that the channel state information is known to the receiver. Also, many communication schemes suitable for data transmission through multiple-antenna wireless channels have been proposed, including Bell Labs Layered Space–Time (BLAST) [17], space–time trellis codes [11], space–time block codes from orthogonal designs [16], and unitary space–time codes [18, 19]. It is worth mentioning that, Alamouti's space-time block code has been established as a part of the W-CDMA and CDMA-2000 standards. In recent articles, [4, 6, 20], STBCs for three, four, five, and eight transmit antennas, with different rates were constructed. These orthogonal designs are with maximal rates.

In this paper, it is intended to present a detailed design, theoretical analysis and investigation of the performance of many classes of OSTBC codes, such as real orthogonal space time block codes for real signal constellations such as PAM signal sets, and complex orthogonal space time block codes for complex signal constellations such as PSK and QAM signal sets. The performance of such systems will be evaluated and simulated using MATLAB.

This paper is organized as follows: section 2 gives a brief background on STBC. Real and complex STBC with general analysis are presented in section 3. In section 4, simulation results are discussed. Finally, section 5 gathers the conclusions and future work.

2. Transmission model of Multiple-Input Multiple-Output channels

When wireless communication systems are using N_t transmit antennas and N_r receive antennas, they are called Multiple-Input Multiple-Output (MIMO) systems. Each signal goes from i^{th} transmit antenna to all receive antennas. Each pair of transmit and receive antennas provide a signal path from transmitter to receiver. Figure 2 shows such a transmission model.

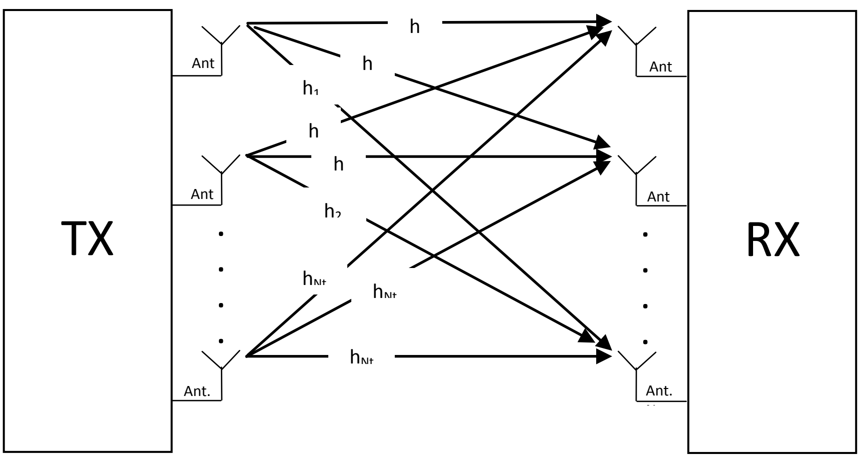


Figure (2): Multiple-Input Multiple-Output system model.

Based on this model the received signal by the m^{th} antenna at time t due to signals transmitted from N_r transmit antennas can be expressed as follows

$$r_{tm} = \sum_{n=1}^{N_t} h_{ij} S_{tn} + n_{tm} \quad (1)$$

Where r_{tm} is the received signal from antenna m at time t, h_{ij} is the channel coefficient, S_{tn} is the transmitted signal from antenna n at time t, and n_{tm} is the noise sample of the receive

antenna m at time t.
In equation 1, the channel is flat fading that means each channel fades independently. Assuming a quasi-static channel, where the channel coefficients are constant over a frame of length T' and change from frame to frame, which is assumed in this paper. In matrix form, the overall transmitted signals from transmit antenna n at time t can be expressed by a $T \times N_t$ matrix as follows

$$S = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1N_t} \\ \vdots & \vdots & \ddots & \vdots \\ S_{T1} & S_{T2} & \cdots & S_{TN_t} \end{pmatrix} \quad (2)$$

Also the channel coefficients are gathered in a matrix form in a $N_t \times N_r$ matrix as follows

$$H = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_r} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_{t1}} & h_{N_{t2}} & \cdots & h_{N_tN_r} \end{pmatrix} \quad (3)$$

The Noise samples matrix is:

$$\mathcal{N} = \begin{pmatrix} n_{11} & n_{12} & \cdots & n_{1N_r} \\ \vdots & \vdots & \ddots & \vdots \\ n_{T1} & n_{T2} & \cdots & n_{TN_r} \end{pmatrix} \quad (4)$$

Therefore, the received signal in a matrix form can be written as follows

$$r = \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1\,N_r} \\ \vdots & \vdots & \ddots & \vdots \\ r_{T1} & r_{T2} & \cdots & r_{TN_r} \end{pmatrix} \quad (5)$$

So, equation (1) can be written in a matrix form as follows [4, 6]

$$r = S.H + \mathcal{N} \quad (6)$$

Another assumption is that the model is quasi-static slow fading such that the noise samples are independent samples of zero-mean circularly symmetric complex Gaussian random variables. This is an Additive White Gaussian Noise (AWGN) assumption for a complex baseband transmission.

3. Design of Orthogonal Space-Time Block Codes

In this section the fundamental design issues of real / complex orthogonal STBCs are presented.

3.1 Design of Real Orthogonal Space-Time Block Codes

This is based on the well-known Radon and Hurwitz theory [6] for designing real orthogonal square and non square matrices.

Definition 1 [6]: A real orthogonal design of size N is an $N \times N$ orthogonal matrix G_N with real entries $x_1, -x_1, x_2, -x_2, \ldots, x_N - x_N$ such that:

$$G_N^T G_N = (x_1^2 + x_2^2 + \cdots + x_N^2) I_N \quad (7)$$

Theorem 1 [6]: A real orthogonal design exists if and only if $N = 2, 4 \; and \; 8$.

The above theorem limits the number of transmit antennas to $N = 2, 4 \; and \; 8$.

Next we refer to the following results which are used to generalize to non square matrices and show the existence of such matrices.

Definition 2 [6]: A generalized real orthogonal design is a $T \times N$ matrix G with real entries $x_1, -x_1, x_2, -x_2, \ldots, x_k - x_k$ such that:

$$G^T . G = k\left(x_1^2 + x_2^2 + \cdots + x_k^2\right) I_N \quad (8)$$

where I_N is the $N \times N$ identity matrix and k is a constant.

Theorem 2 [6]: For any number of transmit antennas, N, there exists a full rate, $R = 1$, real STBC with a block size $T = min[2^{4c+d}]$, where the minimization is over all possible integer values of c and d in the set $\{c \geq 0, d \geq 0 \,|\, 8c + 2^d > N\}$.

3.2 Design of real orthogonal space-time block codes

A real space-time block code is defined by G as a transmission matrix. Given a real constellation with 2^b symbols, for each block of Kb bits, the encoder first picks K symbols $(s_1, s_2, \cdots, s_K)$ from the constellation. Then x_k is replaced by s_k in G to get $S = G(s_1, s_2, \cdots, s_K)$. At time $t = 1,2, \cdots, T$ the $(t, n)^{th}$ element of $S, S_{t,n}$ is transmitted from antenna $n \; for \; n = 1,2, \cdots, N$. The rate of the code is defined by $R = \frac{K}{T}$.

3.3 Design of Complex Orthogonal Space-Time Block Codes

In the previous section all the designs only work for real signal constellations such as PAM signal constellations. So next, the schemes are extended in this section, to complex signal constellations such as PSK and QAM.

Definition 3 [6]: A complex orthogonal design of size N is an $N \times N$ orthogonal matrix G_N with complex entries $x_1, -x_1, x_2, -x_2, \dots, x_N - x_N$ and their conjugates $x_1^*, -x_1^*, x_2^*, \cdots, x_N^*, -x_N^*$ and multiples of these indeterminate variables by $j = \sqrt{-1}$ or –j such that:

$$G_N^H G_N = \left(|x_1|^2 + |x_2|^2 + \cdots + |x_N|^2\right) I_N \tag{9}$$

Complex orthogonal designs exist if and only if $N = 2$.

The next definition and theorem provide a generalization to non-square matrices.

Definition 4 [6]: A generalized complex orthogonal design is a $T \times N$ matrix G with complex entries $x_1, -x_1, x_2, -x_2, \dots, x_k - x_k$ and their conjugates, $x_1^*, -x_1^*, x_2^*, \cdots, x_k^*, -x_k^*$ and multiples of these indeterminate variables by $j = \sqrt{-1}$ or –j such that:

$$G_N^H G_N = k\left(|x_1|^2 + |x_2|^2 + \cdots + |x_N|^2\right) I_N \tag{10}$$

To construct generalized OSTBC codes, we use the following construction method that is based on the above theory of the design of real OSTBC codes.

Construction of generalized complex OSTBC

Given a rate R real orthogonal design with a $T \times N$ transmission matrix S. We note that $K = RT$ symbols are transmitted in each block of T time slots. The conjugate of S is a $T \times N$ denoted by S^* that is derived by replacing s_k with s_k^* in S. We design a $2T \times N$ complex orthogonal design by concatenating S and S^* as follows

$$S_c = \binom{S}{S^*} \tag{11}$$

The resulting complex OSTBC codes, generated using this construction technique are presented in the table in appendix A, for the following number of transmit antennas, $N_t = 2,3,4,5,6,7, and\ 8$

3.4 Maximum-likelihood decoding and maximum ratio combining for OSTBC codes

In this section we review the maximum likelihood (ML) decoding and the maximum ratio combining (MRC) rules for the class of STBC codes considered in this paper.

Based on the theory in [6], the probability distribution function (PDF) of the received signals for a known signal matrix S and channel matrix H, denoted by $f(r|S,H)$, can written as follows

$$f(r|S,H) = \frac{1}{(\pi N_0)^{N_T \times N_T/2}} exp\left\{\frac{-T_r\left[(r-S.H)^H(r-S.H)\right]}{N_0}\right\}$$

$$= \left(\frac{\gamma}{\pi}\right)^{\frac{N_T \times N_T}{2}} exp\{-\gamma T_r[(r-S.H)^H(r-S.H)]\} \tag{12}$$

The Maximum Likelihood (ML) decoding decides in favour of a signal code $\hat{S}$ that maximizes $f(r|S,H)$ given above, which is equivalent to the following minimization problem:

$$\ddot{S} = arg\, min_S\{T_r[(r - S.H)^H.(r - S.H)]\} \quad (13)$$

By expanding equation 13 and noting that $r^H.r$ is independent of the transmitted signal code S, we obtain

$$\ddot{S} = arg\, \min_S\{T_r[r^H.r + H^H.S^H.S.H - H^H.S^H.r - r^H.S.H]\}$$

$$= arg\, min_S\{T_r[H^H.S^H.S.H] - 2Re(Tr[H^H.S^H.r])\} \quad (14)$$

To ease the analysis, it is very common to consider only one receive antenna, denoted as the m^{th} antenna, then to generalize to multiple antennas as shown below.

$$\ddot{S} = arg\, min_S\{\sum_{m=1}^{M}[H_m^H.S^H.S.H_m - 2Re(H_m^H.S^H.r_m)]\} \quad (15)$$

Where H_m is the m^{th} column of H and r_m is the m^{th} column of r. This is known as the Maximum Ratio Combining (MRC), which will be used later on.

So, we consider only one receive antenna the m^{th} antenna ,$M = 1$, the minimization problem reduces to the following

$$\ddot{S} = arg\, min_S\{[H_m^H.S^H.S.H_m - 2Re(H_m^H.S^H.r_m)]\} \quad (16)$$

The above minimization problem, takes different forms for different OSTBC codes. For the presented OSTBC codes, we present these forms in appendix B.

3.5 OSTBC codes for eight transmit antennas MIMO systems

Using the construction technique given in previous section, a complex OSTBC for eight transmit antennas can be obtained from a real OSTBC for eight transmit antennas given by the matrix code S_8. The resulting code is having $T = 16\ and\ K = 8$ with rate $R = 0.5$ given by the matrix S_8 in the table in appendix A.

Maximum Likelihood (ML) Decoding

The received signal by the m^{th} antenna at time t due to signals transmitted from the eight transmit antennas is $r_{tm} = \sum_{n=1}^{n=8} h_{ij} S_{tn} + n_{tm}$, which is written in matrix form as follows

$$\begin{pmatrix} r_{11} & \cdots & r_{1N_r} \\ \vdots & \ddots & \vdots \\ r_{16,1} & \cdots & r_{16,N_r} \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 & -s_8 & s_7 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 & -s_5 & -s_6 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 & s_6 & -s_5 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 & s_3 & s_4 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 & -s_4 & s_3 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 & s_1 & -s_2 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* & s_6^* & s_7^* & s_8^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* & -s_8^* & s_7^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* & s_8^* & -s_5^* & -s_6^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* & s_6^* & -s_5^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* & s_2^* & s_3^* & s_4^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix} \cdot \begin{pmatrix} h_{11} & \cdots & h_{1N_r} \\ \vdots & \ddots & \vdots \\ h_{8,1} & \cdots & h_{8,N_r} \end{pmatrix} + \begin{pmatrix} n_{11} & \cdots & n_{1N_r} \\ \vdots & \ddots & \vdots \\ n_{16,1} & \cdots & n_{16,N_r} \end{pmatrix} \quad (17)$$

The ML decoding algorithm is derived as follows
Step1: Construct the decoding matrix $\Omega_{K\times T} = (h_{k,t})$, with $T = 16\ and\ K = 8$ then obtain its Hermitean transpose $\Omega^H_{T\times K}$ by applying the following rule on the code matrix $S = (s_{t,n})$
For the k^{th} symbol , the t^{th} element of Ω_{mk} is defined by

$$\Omega_{mk}(t) = \begin{cases} \pm h_{nm} & if\ s_{t,n} = \pm s_k \\ \pm h^*_{nm} & if\ s_{t,n} = \pm s^*_k \\ 0 & therwise \end{cases} \quad (18)$$

The decoding matrix for this code is listed in the table in appendix A.

Step 2: From the received vector and its transpose for the m^{th} receive antenna using the following rule [6]:

$$(r'_m)^k(t) = \begin{cases} r^*_m(t) & if\ s^*_k\ or - s^*_k\ exists\ in\ the\ t^{th} row\ of\ S \\ r_m(t) & otherwise \end{cases} \quad (19)$$

We get

$$(r')^T = \begin{pmatrix} r_{1m} & r_{2m} & r_{3m} & r_{4m} & r_{5m} & r_{6m} & r_{7m} & r_{8m} & r^*_{9m} & r^*_{10,m} & r^*_{11,m} & r^*_{12,m} & r^*_{13,m} & r^*_{14,m} & r^*_{15,m} & r^*_{16,m} \end{pmatrix}$$

Step 3: Compute the estimates of the transmitted symbols $\{s_1, s_2, \cdots, s_K\}$ denoted by $\{\widetilde{s_1}, \widetilde{s_2}, \cdots, \widetilde{s_K}\}$ by performing the product $(r')^T.\Omega^H$. These eight estimates are listed in appendix B.

Step 3 : Given the computed estimates $(\widetilde{s_1}\ \ \widetilde{s_2}\ \ \widetilde{s_3}\ \ \widetilde{s_4}\ \widetilde{s_5}\ \widetilde{s_6}\ \widetilde{s_7}\ \widetilde{s_8})$, the ML decoder decides in favor $s_1, s_2, s_3, s_4, s_5, s_6, s_7,$ and s_8 over all possible values of s_1, s_2, $s_3, s_4, s_5, s_6, s_7,$ and s_8 by minimizing the following eight separate cost functions:

$$|\widetilde{s_k} - s_k|^2 + \left(-1 + 2\sum_{n=1}^{n=N_t}\sum_{m=1}^{m=N_r}|h_{nm}|^2\right)|s_k|^2\ , for\ k = 1,2,3,4,5,6,7\ and\ 8 \quad (20)$$

3.6 OSTBC for 7, 6, and 5 transmit antennas MIMO systems

By applying the same steps as above, we obtain the OSTBC codes and their decoding matrices for MIMO systems with 7, 6, and 5 transmit antennas and any number of receive antennas.

1. For seven transmit antennas the resulting code is having $T = 16\ and\ K = 8$ with rate $R = 0.5$ given by the matrix S_7 in the table in appendix A. This code is obtained by deleting the last column in S_7.
2. Similarly, for six transmit antennas the resulting code is having $T = 16\ and\ K = 8$ with rate $R = 0.5$ given by the matrix S_6 in the table in appendix A.
3. Finally, for five transmit antennas the resulting code is having $T = 16\ and\ K = 8$ with rate $R = 0.5$ given by the matrix S_5 in the table in appendix A.

On the receiver side, we follow the same procedure as above, to derive the ML decoding equations. The computed estimates $(\tilde{s_1}\ \tilde{s_2}\ \tilde{s_3}\ \tilde{s_4}\ \tilde{s_5}\ \tilde{s_6}\ \tilde{s_7}\ \tilde{s_8})$, are derived and listed in appendix B. The ML decoder decides in favour of s_1, s_2, s_3 s_4 s_5, s_6, s_7 s_8 over all possible values of s_1, s_2, $s_3, s_4, s_5, s_6, s_7,$ and s_8 by minimizing the cost function given in equation (20), by putting $N_t = 7, 6{,}5$ respectively.

3.7 OSTBC codes for four transmit antennas MIMO systems

In this subsection, we consider the design, analysis, performance evaluation of MIMO systems, with four transmit antennas and N_r receive antennas.

Using the same construction technique, a complex OSTBC for four transmit antennas can be obtained from a real OSTBC for four transmit antennas given by the matrix code S_4. The resulting code is having $T = 8\ and\ K = 4$ with rate $R = 0.5$ given by the matrix S_4 in the table in appendix A.

On the receiver side, we follow the same procedure as above, to derive the ML decoding equations. The computed estimates $(\tilde{s_1}\ \tilde{s_2}\ \tilde{s_3}\ \tilde{s_4})$, are derived and listed in appendix B . The ML decoder decides in favour of s_1, s_2, s_3 s_4 over all possible values of s_1, s_2, s_3, s_4 by minimizing the cost function given in equation (20), by putting $N_t = 4$.

$$|\tilde{s}_k - s_k|^2 + \left(-1 + 2\sum_{n=1}^{n=N_t}\sum_{m=1}^{m=N_r}|h_{nm}|^2\right)|s_k|^2, for\ k = 1,2,3\ and\ 4 \qquad (21)$$

3.8 OSTBC for 3 and 2 transmit antennas MIMO systems

In this section, we consider the design, analysis, performance evaluation of MIMO systems, with three transmit antennas and N_r receive antennas.

Using the same construction technique, a complex OSTBC for four transmit antennas can be obtained from the complex OSTBC for four transmit antennas given by the matrix code S_4

by removing the last column. The resulting code is having $T = 8 \text{ and } K = 4$ with rate $R = 0.5$ given by the matrix S_3 in the table in appendix A.
On the receiver side, we follow the same procedure as above, to derive the ML decoding equations. The computed estimates $(\tilde{s_1}\ \tilde{s_2}\ \tilde{s_3}\ \tilde{s_4})$, are derived and listed in appendix B .
The ML decoder decides in favour of s_1, s_2, $s_3\ s_4$ over all possible values of s_1, s_2, s_3, s_4 by minimizing the cost function given in equation (20), by putting $N_t = 3$.

$$|\tilde{s_k} - s_k|^2 + \left(-1 + 2\sum_{n=1}^{n=N_t}\sum_{m=1}^{m=N_r}|h_{nm}|^2\right)|s_k|^2, for\ k = 1,2,3\ and\ 4 \qquad (22)$$

For two transmit antennas, it was shown [6], the complex orthogonal design exits only for $N = 2$, which is given by the following matrix

$$G_2 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \qquad (23)$$

The ML decoder decides in favour of s_1, s_2 over all possible values of s_1, s_2, by minimizing the cost function given in equation (20), by putting $N_t = 2$ for $k = 1 \text{ and } 2$.

4. Simulation

This section deals with performance analysis through simulations of OSTBC codes developed in last section. To compare the performance of OSTBC codes for various MIMO systems with any number of transmits antennas and any numbers of receiving antennas, extensive simulation scenarios have been performed using MATLAB. The OSTBC codes with different parameters have been considered. The communication system model used in this simulation is consisting of three major elements which are the transmitter, the channel and the receiver. On the transmitter side we consider the OSTBC encoder, on the receiver side the ML decoder is implemented and simulated as derived in last section. As stated earlier, it is assumed that the MIMO channel behaves in a “quasi-static” fashion, i.e. the channel varies randomly between burst to burst, but fixed within a transmission. The used channel model is Rayleigh fading channel because it is considered the preferred statistical model for multipath channels where there is no direct path between the transmitter and the receiver [5]. This channel model has independent identically distributed (i.i.d.) complex, zero mean, unit variance channel elements and is given by [4]:

$$h_{ij} = \frac{1}{\sqrt{2}}\left(\text{Normal}(0,1) + \sqrt{-1}.\ \text{Normal}(0,1)\right) \qquad (24)$$

The performances of OSTBC schemes are studied in terms of the Bit Error Rate (BER) and Signal to Noise Ratio (SNR). The environment of simulation is MATLAB which is a powerful tool for mathematical calculation and system simulation. A random sequence generator is used for producing source data. The methods of modulations chosen are QPSK, 16QAM, 64QAM, and 128QAM with gray scale mapping. These were done with varying the number of transmit antennas. The performances of the bit error rate (BER) for OSTBC with different numbers of transmit and receive antennas are shown in different figures depending on the modulation schemes used and number of transmit antennas. In these simulations, 10^4 blocks of symbols are simulated until at least 100 bit errors are obtained. The simulation is stopped when the SNR reached 40dB or after simulating 10^4 blocks without errors.

5. Result and analysis

In this section we present the simulation results and the performance of OSTBC codes for MIMO systems with four, three and two transmit antennas and $N_r = 1, 2, \text{and } 4$ receive antennas for different modulation schemes.

Case 1: For four transmit and N_r receive antennas

In this section, we provide and compare the simulation results for the implemented OSTBC-MIMO system. For this case, we consider the two codes given by G_4 & H_4 (see appendix A) In this case the parameters are $N_t = 4$, N_r is fixed to 1 and for the following modulations QPSK, 16QAM, 64QAM, 128QAM using the code denoted by G_4.
Figure (3): shows the simulated BER versus SNR for $N_r = 1$.

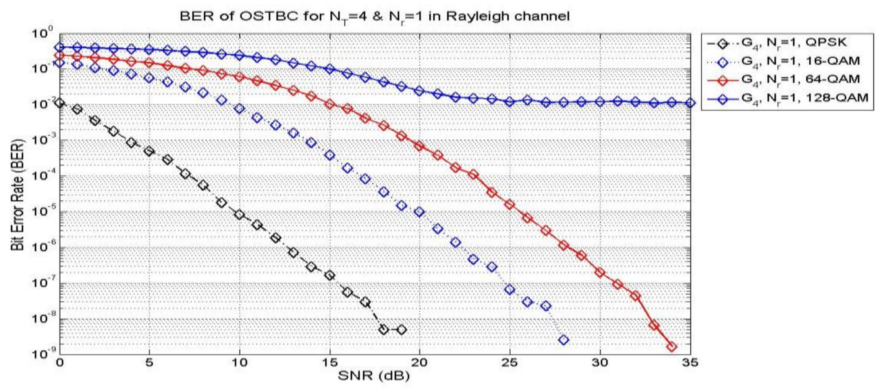


Figure (3): Simulated BER vs. SNR for $N_r = 1$ and $N_t = 4$

As observed from the figure 3, QPSK gives the lowest value of BER at SNR = 2dB, and also at SNR = 15dB. As expected, the performance in terms of the Bit error rate improves as we go for less number of signal points in the constellations, which means less number of bits per symbol (Size of the constellation). From figure 3, it is observed that QPSK (two bits/symbol) is better than 16QAM (four bits/symbol) by approximately 9 dB at BER equal to 10^{-3} . Similarly 16QAM is better than 64QAM.

Next, we compare the performance of the two codes given by G_4 & H_4 . The parameters are $N_t = 4$, N_r is fixed to 1,2, and 4 and using the 16QAM modulation .

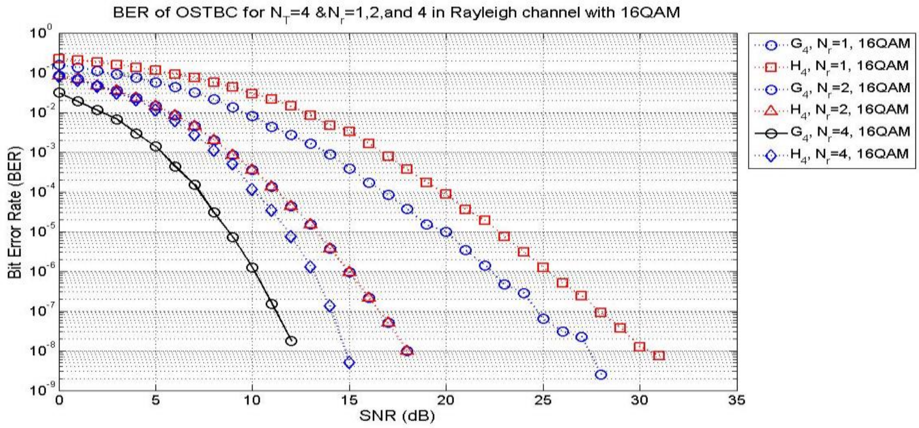


Figure (4): Simulated BER vs. SNR for $N_r = 1,2$ &4 at $N_t = 4$

By considering a fixed code for example G_4 , to study the effect of number of receive antennas, we observe from figure 4, at BER 10^{-5} the gain obtained by using $N_r = 2$ over $N_r = 1$ is approximately 7 dB, while the gain obtained by using $N_r = 4$ over $N_r = 2$ is approximately 2 dB. The same result is obtained for the code H_4 .

To compare the performance between the codes G_4 & H_4 for fixed $N_r = 1$, we observe from figure 4, that at low SNR G_4 performs better than H_4 by approximately 1 dB, while at high SNR G_4 performs better than H_4 by approximately 3 dB. For a fixed $N_r = 4$, it is observed from figure 4, that G_4 performs better than H_4 by approximately 3 dB. While for fixed $N_r = 2$, it is observed from figure 4, that G_4 and H_4 performs almost equally for values of SNR.

Case 2: For three transmit and N_r receive antennas

In this section, we provide and compare the simulation results for the implemented OSTBC-MIMO system for three transmit antennas. For this case, we used two codes given by G_3 & H_3 (see appendix A).

In this case, we compare the performance of the two codes given by G_3 & H_3 . The parameters are $N_t = 3$, N_r is fixed to 1,2, and 4 and using the 16QAM modulation .

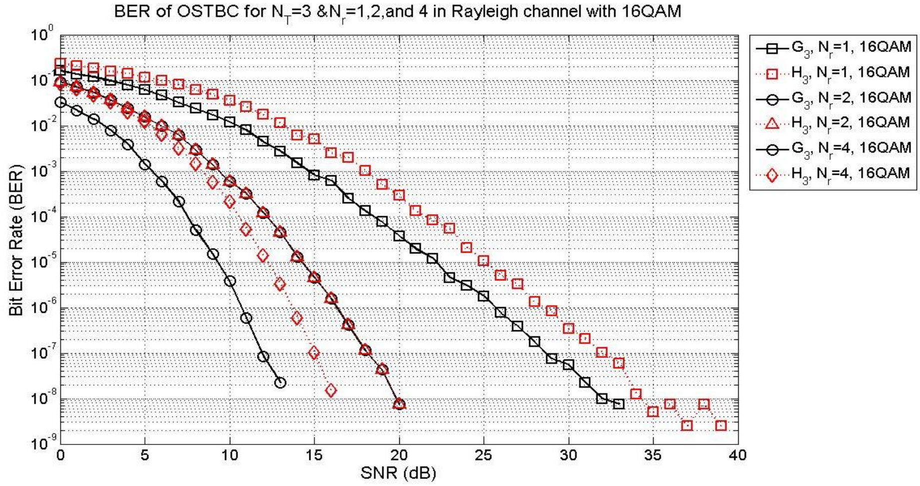


Figure (5): Simulated BER vs. SNR for $N_r = 1,2\ \&4$ at $N_t = 3$

By considering a fixed code for example G_3 , to study the effect of number of receive antennas, we observe from figure 6, at BER 10^{-5} the gain obtained by using $N_r = 2$ over $N_r = 1$ is approximately 7 dB, while the gain obtained by using $N_r = 4$ over $N_r = 2$ is approximately 2 dB. The same result is obtained for the code H_3 .

For fixed $N_r = 1$, to compare the performance between the codes $G_3\ \&\ H_3$, we observe from figure 4.16, that at low SNR G_3 performs better than H_3 by approximately 1 dB, while at high SNR G_3 performs better than H_3 by approximately 3 dB. For fixed $N_r = 4$, it is observed from figure 4.16, that G_3 performs better than H_3 by approximately 3 dB. While for fixed $N_r = 2$, it is observed from figure 6, that G_3 and H_3 performs almost equally for values of SNR.

Case 3: For two transmit and N_r receive antennas

In this section, we provide and compare the simulation results for the implemented OSTBC-MIMO system. For this case, we used two codes given by G_2 . In this case, the parameters are $N_t = 2$, N_r is varying as 1,2, and 4 and for the following modulations 16QAM.

Figure (6) shows the simulated BER versus SNR for 16QAM.

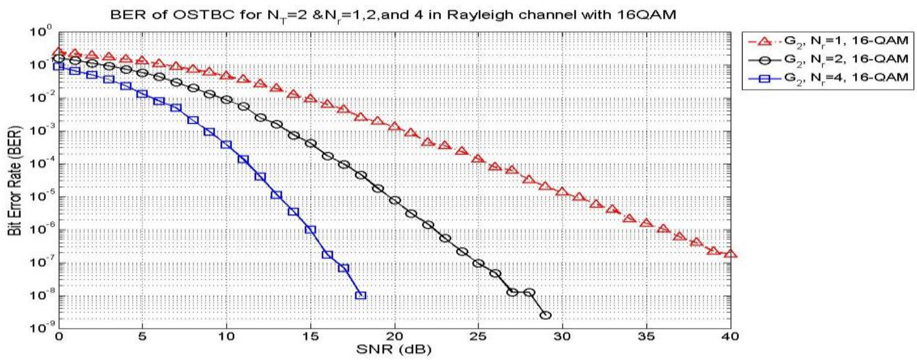


Figure (6): Simulated BER vs. SNR for $N_r = 1,2 \& 4$ at $N_t = 2$

As observed from the figure 7, four receive antennas gives the lowest value of BER at SNR = 2dB, and also at SNR = 8dB. This result is in accordance with the theory presented. So we conclude that the performance in terms of the Bit error rate improves as we go for more number of receive antennas. From figure 6, the gain by using $N_r = 2$ over $N_r = 1$ is approximately 6.2dB, while the gain by using the $N_r = 4$ over $N_r = 2$ is approximately 9.8dB.

6. Conclusion

This paper provided the detailed design of real and complex OSTBC codes to be used with real signal set constellation such PAM and complex signal constellation such as PSK and QAM. Real and complex OSTBC codes for MIMO systems with two, three, four, five, six, seven and eight transmit antennas and any number of receive antennas. Simple linear processing ML decoders were derived and presented. The used channel is Rayleigh fading channel MIMO and assumed to behave in a "quasi-static" fashion. The performances of OSTBC schemes were evaluated and compared in terms of the Bit Error Rate (BER) and Signal to Noise Ratio (SNR). The environment of simulation was MATLAB which is a powerful tool for mathematical calculation and system simulation. The methods of modulations chosen were QPSK, 16QAM, 64QAM, and 128QAM with gray scale mapping.

7. References

[1] Elzinati, M., Space-time Block Coding for Wireless Communications. 2008.
[2] Paul, G.M.K.O.O., " Performance Evaluation of LTE Downlink with MIMO Techniques, in Engineering ". 2010.

[3] V, S., " High-Rate and Information-Lossless Space-Time Block Codes from Crossed-Product Algebras, in Electrical Communication Engineering ". 2004, Indian Institute of Science.
[4] Cort´es-Pe˜na, L.M., " MIMO Space-Time Block Coding (STBC): Simulations and Results ". DESIGN PROJECT: PERSONAL AND MOBILE COMMUNICATIONS, 2009: p. 8.
[5] Hampton, J.R., " Introduction to MIMO Communications ". 2014: Cambridge University Press.
[6] Jafarkhani, H., " Space - Time Coding Theory and Practice ". 2005: Cambridge University Press.
[7] Salehi, J.G.P.a.M., " Digital Communications ". 2008: McGraw-Hill.
[8] Wittneben, A., " A new bandwidth efficient transmit antenna modulation diversity scheme for linear digital modulation ". Proc. IEEE International Conf. Communications 1993: p. 1630–1634.
[9] Wittneben, A., " Base station modulation diversity for digital simulcast ". Proc. IEEE Vehicular Technology Conf., 1991: p. 848–853.
[10] Winters, N.S.a.J.H., " Two signaling schemes for improving the error performance of FDD transmission systems using the transmitter antenna diversity ". Proc. IEEE Vehicular Technology Conf., 1993: p. 508–511.
[11] V. Tarokh, N.S., and A. R. Calderbank, " Space-time codes for high data rate wireless communication: Performance criteria and code construction ". IEEE Trans. Inform. Theory, 1998: p. 744-765.
[12] Alamouti, S., " A Simple Transmit Diversity Technique for Wireless Communications ". IEEE-JSAC, 1998. 16(8): p. 1451-1458.
[13] Taha, Z.Q., " Efficient Decoding for Extended Alamouti Space-Time Block code ". International Journal of Distributed and Parallel Systems (IJDPS), 2011. 2(2): p. 96-103.
[14] A. Idris, K.D., and S. K. Syed Yusof, " Performance of Linear Maximum Likelihood Alamouti Decoder with Diversity Techniques ". Proceedings of the World Congress on Engineering, 2011. 2.
[15] V. Tarokh, H.J., and A. R. Calderbank, " The application of orthogonal designs to wireless communication ". Proc. IEEE Information Theory Workshop: p. 46–47.
[16] V. Tarokh, H.J., and A. Calderbank, " Space–Time Block Codes from Orthogonal Designs ". IEEE TRANSACTIONS ON INFORMATION THEORY, 1999. 45: p. 12.
[17] Foschini, G.J., " Layered space-time architecture for wireless communication in fading environment when using multi-element antennas ". Bell Labs Tech. J., 1996. 1(2): p. 41-59.
[18] A. Shokrollahi, B.H., B. M. Hochwald, and W. Sweldens, " Representation theory for high-rate multiple-antenna code design ". IEEE Trans. Commun., 2001. 47: p. 2335-2367.
[19] Marzetta, B.M.H.a.T.L., " Unitary space-time modulation for multiple-antenna communication in Rayleigh flat fading ". IEEE Trans. Inform. Theory, 2000. 46: p. 543-564.
[20] Liang, X.B., " Orthogonal Designs with Maximal Rates ". IEEE Trans. Inform. Theory, 2003. 49: p. 2468-2503.

Appendices

Appendix A: Transmission matrix and decoding matrix for OSTBC codes

S. No.	N_t	Transmission Matrix $S_{T\times N_t}$	Hermitian of Decoding Matrix
1	8	$S_8 = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 & -s_8 & s_7 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 & -s_5 & -s_6 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 & s_6 & -s_5 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 & s_3 & s_4 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 & -s_4 & s_3 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 & s_1 & -s_2 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* & s_6^* & s_7^* & s_8^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* & -s_8^* & s_7^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* & s_8^* & -s_5^* & -s_6^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* & s_6^* & -s_5^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* & s_2^* & s_3^* & s_4^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix}$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & h_{4m}^* & h_{5m}^* & h_{6m}^* & h_{7m}^* & h_{8m}^* \\ h_{2m}^* & -h_{1m}^* & -h_{4m}^* & h_{3m}^* & -h_{6m}^* & h_{5m}^* & h_{8m}^* & -h_{7m}^* \\ h_{3m}^* & h_{4m}^* & -h_{1m}^* & -h_{2m}^* & -h_{7m}^* & -h_{8m}^* & h_{5m}^* & h_{6m}^* \\ h_{4m}^* & -h_{3m}^* & h_{2m}^* & -h_{1m}^* & -h_{8m}^* & h_{7m}^* & -h_{6m}^* & h_{5m}^* \\ h_{5m}^* & h_{6m}^* & h_{7m}^* & h_{8m}^* & -h_{1m}^* & -h_{2m}^* & -h_{3m}^* & -h_{4m}^* \\ h_{6m}^* & -h_{5m}^* & h_{8m}^* & -h_{7m}^* & h_{2m}^* & -h_{1m}^* & h_{4m}^* & -h_{3m}^* \\ h_{7m}^* & -h_{8m}^* & -h_{5m}^* & h_{6m}^* & h_{3m}^* & -h_{4m}^* & -h_{1m}^* & h_{2m}^* \\ h_{8m}^* & h_{7m}^* & -h_{6m}^* & -h_{5m}^* & h_{4m}^* & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1,m} & h_{2,m} & h_{3,m} & h_{4,m} & h_{5,m} & h_{6,m} & h_{7,m} & h_{8,m} \\ h_{2,m} & -h_{1,m} & -h_{4,m} & h_{3,m} & -h_{6,m} & h_{5,m} & h_{8,m} & -h_{7,m} \\ h_{3,m} & h_{4,m} & -h_{1,m} & -h_{2,m} & -h_{7,m} & -h_{8,m} & h_{5,m} & h_{6,m} \\ h_{4,m} & -h_{3,m} & h_{2,m} & -h_{1,m} & -h_{8,m} & h_{7,m} & -h_{6,m} & h_{5,m} \\ h_{5,m} & h_{6,m} & h_{7,m} & h_{8,m} & -h_{1,m} & -h_{2,m} & -h_{3,m} & -h_{4,m} \\ h_{6,m} & -h_{5,m} & h_{8,m} & -h_{7,m} & h_{2,m} & -h_{1,m} & h_{4,m} & -h_{3,m} \\ h_{7,m} & -h_{8,m} & -h_{5,m} & h_{6,m} & h_{3,m} & -h_{4,m} & -h_{1,m} & h_{2,m} \\ h_{8,m} & h_{7,m} & -h_{6,m} & -h_{5,m} & h_{4,m} & h_{3,m} & -h_{2,m} & -h_{1,m} \end{pmatrix}$
2	7	$S_7 = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 & -s_8 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 & -s_5 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 & s_6 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 & s_3 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 & -s_4 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 & s_1 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* & s_6^* & s_7^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* & -s_8^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* & s_8^* & -s_5^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* & s_6^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* & s_2^* & s_3^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* & -s_4^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* & s_4^* & s_1^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* & s_2^* \end{pmatrix}$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & h_{4m}^* & h_{5m}^* & h_{6m}^* & h_{7m}^* & 0 \\ h_{2m}^* & -h_{1m}^* & -h_{4m}^* & h_{3m}^* & -h_{6m}^* & h_{5m}^* & 0 & -h_{7m}^* \\ h_{3m}^* & h_{4m}^* & -h_{1m}^* & -h_{2m}^* & -h_{7m}^* & 0 & h_{5m}^* & h_{6m}^* \\ h_{4m}^* & -h_{3m}^* & h_{2m}^* & -h_{1m}^* & 0 & h_{7m}^* & -h_{6m}^* & h_{5m}^* \\ h_{5m}^* & h_{6m}^* & h_{7m}^* & 0 & -h_{1m}^* & -h_{2m}^* & -h_{3m}^* & -h_{4m}^* \\ h_{6m}^* & -h_{5m}^* & 0 & -h_{7m}^* & h_{2m}^* & -h_{1m}^* & h_{4m}^* & -h_{3m}^* \\ h_{7m}^* & 0 & -h_{5m}^* & h_{6m}^* & h_{3m}^* & -h_{4m}^* & -h_{1m}^* & h_{2m}^* \\ 0 & h_{7m}^* & -h_{6m}^* & -h_{5m}^* & h_{4m}^* & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1,m} & h_{2,m} & h_{3,m} & h_{4,m} & h_{5,m} & h_{6,m} & h_{7,m} & 0 \\ h_{2,m} & -h_{1,m} & -h_{4,m} & h_{3,m} & -h_{6,m} & h_{5,m} & 0 & -h_{7,m} \\ h_{3,m} & h_{4,m} & -h_{1,m} & -h_{2,m} & -h_{7,m} & 0 & h_{5,m} & h_{6,m} \\ h_{4,m} & -h_{3,m} & h_{2,m} & -h_{1,m} & 0 & h_{7,m} & -h_{6,m} & h_{5,m} \\ h_{5,m} & h_{6,m} & h_{7,m} & 0 & -h_{1,m} & -h_{2,m} & -h_{3,m} & -h_{4,m} \\ h_{6,m} & -h_{5,m} & 0 & -h_{7,m} & h_{2,m} & -h_{1,m} & h_{4,m} & -h_{3,m} \\ h_{7,m} & 0 & -h_{5,m} & h_{6,m} & h_{3,m} & -h_{4,m} & -h_{1,m} & h_{2,m} \\ 0 & h_{7,m} & -h_{6,m} & -h_{5,m} & h_{4,m} & h_{3,m} & -h_{2,m} & -h_{1,m} \end{pmatrix}$

3	6	$S_6 = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* & s_6^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* & s_8^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* & s_2^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* & s_4^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* \end{pmatrix}$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & h_{4m}^* & h_{5m}^* & h_{6m}^* & 0 & 0 \\ h_{2m}^* & -h_{1m}^* & -h_{4m}^* & h_{3m}^* & -h_{6m}^* & h_{5m}^* & 0 & 0 \\ h_{3m}^* & h_{4m}^* & -h_{1m}^* & -h_{2m}^* & 0 & 0 & h_{5m}^* & h_{6m}^* \\ h_{4m}^* & -h_{3m}^* & h_{2m}^* & -h_{1m}^* & 0 & 0 & -h_{6m}^* & h_{5m}^* \\ h_{5m}^* & h_{6m}^* & 0 & 0 & -h_{1m}^* & -h_{2m}^* & -h_{3m}^* & -h_{4m}^* \\ h_{6m}^* & -h_{5m}^* & 0 & 0 & h_{2m}^* & -h_{1m}^* & h_{4m}^* & -h_{3m}^* \\ 0 & 0 & -h_{5m}^* & h_{6m}^* & h_{3m}^* & -h_{4m}^* & -h_{1m}^* & h_{2m}^* \\ 0 & 0 & -h_{6m}^* & -h_{5m}^* & h_{4m}^* & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1,m} & h_{2,m} & h_{3,m} & h_{4,m} & h_{5,m} & h_{6,m} & 0 & 0 \\ h_{2,m} & -h_{1,m} & -h_{4,m} & h_{3,m} & -h_{6,m} & h_{5,m} & 0 & 0 \\ h_{3,m} & h_{4,m} & -h_{1,m} & -h_{2,m} & 0 & 0 & h_{5,m} & h_{6,m} \\ h_{4,m} & -h_{3,m} & h_{2,m} & -h_{1,m} & 0 & 0 & -h_{6,m} & h_{5,m} \\ h_{5,m} & h_{6,m} & 0 & 0 & -h_{1,m} & -h_{2,m} & -h_{3,m} & -h_{4,m} \\ h_{6,m} & -h_{5,m} & 0 & 0 & h_{2,m} & -h_{1,m} & h_{4,m} & -h_{3,m} \\ 0 & 0 & -h_{5,m} & h_{6,m} & h_{3,m} & -h_{4,m} & -h_{1,m} & h_{2,m} \\ 0 & 0 & -h_{6,m} & -h_{5,m} & h_{4,m} & h_{3,m} & -h_{2,m} & -h_{1,m} \end{pmatrix}$
	5	$S_5 = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* \end{pmatrix}$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & h_{4m}^* & h_{5m}^* & 0 & 0 & 0 \\ h_{2m}^* & -h_{1m}^* & -h_{4m}^* & h_{3m}^* & 0 & h_{5m}^* & 0 & 0 \\ h_{3m}^* & h_{4m}^* & -h_{1m}^* & -h_{2m}^* & 0 & 0 & h_{5m}^* & 0 \\ h_{4m}^* & -h_{3m}^* & h_{2m}^* & -h_{1m}^* & 0 & 0 & 0 & h_{5m}^* \\ h_{5m}^* & 0 & 0 & 0 & -h_{1m}^* & -h_{2m}^* & -h_{3m}^* & -h_{4m}^* \\ 0 & -h_{5m}^* & 0 & 0 & h_{2m}^* & -h_{1m}^* & h_{4m}^* & -h_{3m}^* \\ 0 & 0 & -h_{5m}^* & 0 & h_{3m}^* & -h_{4m}^* & -h_{1m}^* & h_{2m}^* \\ 0 & 0 & 0 & -h_{5m}^* & h_{4m}^* & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1,m} & h_{2,m} & h_{3,m} & h_{4,m} & h_{5,m} & 0 & 0 & 0 \\ h_{2,m} & -h_{1,m} & -h_{4,m} & h_{3,m} & 0 & h_{5,m} & 0 & 0 \\ h_{3,m} & h_{4,m} & -h_{1,m} & -h_{2,m} & 0 & 0 & h_{5,m} & 0 \\ h_{4,m} & -h_{3,m} & h_{2,m} & -h_{1,m} & 0 & 0 & 0 & h_{5,m} \\ h_{5,m} & 0 & 0 & 0 & -h_{1,m} & -h_{2,m} & -h_{3,m} & -h_{4,m} \\ 0 & -h_{5,m} & 0 & 0 & h_{2,m} & -h_{1,m} & h_{4,m} & -h_{3,m} \\ 0 & 0 & -h_{5,m} & 0 & h_{3,m} & -h_{4,m} & -h_{1,m} & h_{2,m} \\ 0 & 0 & 0 & -h_{5,m} & h_{4,m} & h_{3,m} & -h_{2,m} & -h_{1,m} \end{pmatrix}$
	4	$G = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2 & s_1 & -s_4 & s_3 \\ -s_3 & s_4 & s_1 & -s_2 \\ -s_4 & -s_3 & s_2 & s_1 \\ s_1^* & s_2^* & s_3^* & s_4^* \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & s_4^* & s_1^* & -s_2^* \\ -s_4^* & -s_3^* & s_2^* & s_1^* \end{pmatrix} \triangleq G_4$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & h_{4m}^* \\ h_{2m}^* & -h_{1m}^* & h_{4m}^* & -h_{3m}^* \\ h_{3m}^* & -h_{4m}^* & -h_{1m}^* & h_{2m}^* \\ h_{4m}^* & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1m} & h_{2m} & h_{3m} & h_{4m} \\ h_{2m} & -h_{1m} & h_{4m} & -h_{3m} \\ h_{3m} & -h_{4m} & -h_{1m} & h_{2m} \\ h_{4m} & h_{3m} & -h_{2m} & -h_{1m} \end{pmatrix}$

	$H_4 = \begin{pmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{-s_1-s_1^*+s_2-s_2^*}{2} & \frac{-s_2-s_2^*+s_1-s_1^*}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{s_2+s_2^*+s_1-s_1^*}{2} & \frac{s_1+s_1^*+s_2-s_2^*}{2} \end{pmatrix}$	
3	$G_3 = \begin{pmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{pmatrix}$ $H_3 = \begin{pmatrix} s_1 & s_2 & \frac{s_3}{\sqrt{2}} \\ -s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} \\ \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{-s_1-s_1^*+s_2-s_2^*}{2} \\ \frac{s_3^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{s_2+s_2^*+s_1-s_1^*}{2} \end{pmatrix}$	$\begin{pmatrix} h_{1m}^* & h_{2m}^* & h_{3m}^* & 0 \\ h_{2m}^* & -h_{1m}^* & 0 & -h_{3m}^* \\ h_{3m}^* & 0 & -h_{1m}^* & h_{2m}^* \\ 0 & h_{3m}^* & -h_{2m}^* & -h_{1m}^* \\ h_{1m} & h_{2m} & h_{3m} & 0 \\ h_{2m} & -h_{1m} & 0 & -h_{3m} \\ h_{3m} & 0 & -h_{1m} & h_{2m} \\ 0 & h_{3m} & -h_{2m} & -h_{1m} \end{pmatrix}$
2	$G_2 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}$	$\begin{pmatrix} h_{1,m}^* & h_{2,m}^* \\ h_{2,m} & -h_{1,m}^* \end{pmatrix}$

Appendix B: Derivation of the decoding formulae for $N_t = 8, 7, 6, 5, 4, 3$ and 2 transmit antennas MIMO systems.

B.1: $N_t = 8$

$$\tilde{s}_1 = \sum_{m=1}^{N_r} r_{1m}h_{1m}^* + r_{2m}h_{2m}^* + r_{3m}h_{3m}^* + r_{4m}h_{4m}^* + r_{5m}h_{5m}^* + r_{6m}h_{6m}^* + r_{7m}h_{7m}^* + r_{8m}h_{8m}^* + r_{9m}^*h_{1m} + r_{10m}^*h_{2m} + r_{11m}^*h_{3m} + r_{12m}^*h_{4m} + r_{13m}^*h_{5m} + r_{14m}^*h_{6m} + r_{15m}^*h_{7m} + r_{16m}^*h_{8m}$$

$$\tilde{s}_2 = \sum_{m=1}^{N_r} r_{1m}h_{2m}^* - r_{2m}h_{1m}^* + r_{3m}h_{4m}^* - r_{4m}h_{3m}^* + r_{5m}h_{6m}^* - r_{6m}h_{5m}^* - r_{7m}h_{8m}^* + r_{8m}h_{7m}^* + r_{9m}^*h_{2m} - r_{10m}^*h_{1m} + r_{11m}^*h_{4m} - r_{12m}^*h_{3m} + r_{13m}^*h_{6m} - r_{14m}^*h_{5m} - r_{15m}^*h_{8m} + r_{16m}^*h_{7m},$$

$$\tilde{s}_3 = \sum_{m=1}^{N_r} r_{1m}h_{3m}^* - r_{2m}h_{4m}^* - r_{3m}h_{1m}^* + r_{4m}h_{2m}^* + r_{5m}h_{7m}^* + r_{6m}h_{8m}^* - r_{7m}h_{5m}^* - r_{8m}h_{6m}^* + r_{9m}^*h_{3m} - r_{10m}^*h_{4m} + r_{11m}^*h_{1m} + r_{12m}^*h_{2m} + r_{13m}^*h_{7m} + r_{14m}^*h_{8m} - r_{15m}^*h_{5m} - r_{16m}^*h_{6m}$$

$$\tilde{s}_4 = \sum_{m=1}^{N_r} r_{1m}h_{4m}^* + r_{2m}h_{3m}^* - r_{3m}h_{2m}^* - r_{4m}h_{1m}^* + r_{5m}h_{8m}^* - r_{6m}h_{7m}^* + r_{7m}h_{6m}^* - r_{8m}h_{5m}^* + r_{9m}^*h_{4m} + r_{10m}^*h_{3m} - r_{11m}^*h_{2m} - r_{12m}^*h_{1m} + r_{13m}^*h_{8m} - r_{14m}^*h_{7m} + r_{15m}^*h_{6m} - r_{16m}^*h_{5m}$$

$$\tilde{s}_5 = \sum_{m=1}^{N_r} r_{1m}h_{5m}^* - r_{2m}h_{6m}^* - r_{3m}h_{7m}^* - r_{4m}h_{8m}^* - r_{5m}h_{1m}^* + r_{6m}h_{2m}^* + r_{7m}h_{3m}^* + r_{8m}h_{4m}^* + r_{9m}^*h_{5m} - r_{10m}^*h_{6m} - r_{11m}^*h_{7m} - r_{12m}^*h_{8m} - r_{13m}^*h_{1m} + r_{14m}^*h_{2m} + r_{15m}^*h_{3m} + r_{16m}^*h_{4m}$$

$$\tilde{s}_6 = \sum_{m=1}^{N_r} r_{1m}h_{6m}^* + r_{2m}h_{5m}^* - r_{3m}h_{8m}^* + r_{4m}h_{7m}^* - r_{5m}h_{2m}^* - r_{6m}h_{1m}^* - r_{7m}h_{4m}^* + r_{8m}h_{3m}^* + r_{9m}^*h_{6m} + r_{10m}^*h_{5m} - r_{11m}^*h_{8m} + r_{12m}^*h_{7m} - r_{13m}^*h_{2m} - r_{14m}^*h_{1m} - r_{15m}^*h_{4m} + r_{16m}^*h_{3m}$$

$$\tilde{s}_7 = \sum_{m=1}^{N_r} r_{1m}h_{7m}^* + r_{2m}h_{8m}^* + r_{3m}h_{5m}^* - r_{4m}h_{6m}^* - r_{5m}h_{3m}^* - r_{6m}h_{4m}^* - r_{7m}h_{1m}^* - r_{8m}h_{2m}^* + r_{9m}^*h_{7m} + r_{10m}^*h_{8m} + r_{11m}^*h_{5m} - r_{12m}^*h_{6m} - r_{13m}^*h_{3m} + r_{14m}^*h_{4m} - r_{15m}^*h_{1m} - r_{16m}^*h_{2m}$$

$$\tilde{s}_8 = \sum_{m=1}^{N_r} r_{1m}h_{8m}^* - r_{2m}h_{7m}^* + r_{3m}h_{6m}^* + r_{4m}h_{5m}^* - r_{5m}h_{4m}^* - r_{6m}h_{3m}^* + r_{7m}h_{2m}^* - r_{8m}h_{1m}^* + r_{9m}^*h_{8m} - r_{10m}^*h_{7m} + r_{11m}^*h_{6m} + r_{12m}^*h_{5m} - r_{13m}^*h_{4m} - r_{14m}^*h_{3m} + r_{15m}^*h_{2m} - r_{16m}^*h_{1m}$$

B.2: $N_t = 7$

$\tilde{s}_1 = \sum_{m=1}^{N_r} r_{1m}h_{1m}^* + r_{2m}h_{2m}^* + r_{3m}h_{3m}^* + r_{4m}h_{4m}^* + r_{5m}h_{5m}^* + r_{6m}h_{6m}^* + r_{7m}h_{7m}^* + r_{9m}^*h_{1m} + r_{10m}^*h_{2m} + r_{11m}^*h_{3m} + r_{12m}^*h_{4m} + r_{13m}^*h_{5m} + r_{14m}^*h_{6m} + r_{15m}^*h_{7m}$,

$\tilde{s}_2 = \sum_{m=1}^{N_r} r_{1m}h_{2m}^* - r_{2m}h_{1m}^* + r_{3m}h_{4m}^* - r_{4m}h_{3m}^* + r_{5m}h_{6m}^* - r_{6m}h_{5m}^* + r_{8m}h_{7m}^* + r_{9m}^*h_{2m} - r_{10m}^*h_{1m} + r_{11m}^*h_{4m} - r_{12m}^*h_{3m} + r_{13m}^*h_{6m} - r_{14m}^*h_{5m} + r_{16m}^*h_{7m}$,

$\tilde{s}_3 = \sum_{m=1}^{N_r} r_{1m}h_{3m}^* - r_{2m}h_{4m}^* - r_{3m}h_{1m}^* + r_{4m}h_{2m}^* + r_{5m}h_{7m}^* - r_{7m}h_{5m}^* - r_{8m}h_{6m}^* + r_{9m}^*h_{3m} - r_{10m}^*h_{4m} + r_{11m}^*h_{1m} + r_{12m}^*h_{2m} + r_{13m}^*h_{7m} - r_{15m}^*h_{5m} - r_{16m}^*h_{6m}$,

$\tilde{s}_4 = \sum_{m=1}^{N_r} r_{1m}h_{4m}^* + r_{2m}h_{3m}^* - r_{3m}h_{2m}^* - r_{4m}h_{1m}^* - r_{6m}h_{7m}^* + r_{7m}h_{6m}^* - r_{8m}h_{5m}^* + r_{9m}^*h_{4m} + r_{10m}^*h_{3m} - r_{11m}^*h_{2m} - r_{12m}^*h_{1m} - r_{14m}^*h_{7m} + r_{15m}^*h_{6m} - r_{16m}^*h_{5m}$,

$\tilde{s}_5 = \sum_{m=1}^{N_r} r_{1m}h_{5m}^* - r_{2m}h_{6m}^* - r_{3m}h_{7m}^* - r_{5m}h_{1m}^* + r_{6m}h_{2m}^* + r_{7m}h_{3m}^* + r_{8m}h_{4m}^* + r_{9m}^*h_{5m} - r_{10m}^*h_{6m} - r_{11m}^*h_{7m} - r_{13m}^*h_{1m} + r_{14m}^*h_{2m} + r_{15m}^*h_{3m} + r_{16m}^*h_{4m}$,

$\tilde{s}_6 = \sum_{m=1}^{N_r} r_{1m}h_{6m}^* + r_{2m}h_{5m}^* + r_{4m}h_{7m}^* - r_{5m}h_{2m}^* - r_{6m}h_{1m}^* - r_{7m}h_{4m}^* + r_{8m}h_{3m}^* + r_{9m}^*h_{6m} + r_{10m}^*h_{5m} + r_{12m}^*h_{7m} - r_{13m}^*h_{2m} - r_{14m}^*h_{1m} - r_{15m}^*h_{4m} + r_{16m}^*h_{3m}$,

$\tilde{s}_7 = \sum_{m=1}^{N_r} r_{1m}h_{7m}^* + r_{3m}h_{5m}^* - r_{4m}h_{6m}^* - r_{5m}h_{3m}^* - r_{6m}h_{4m}^* - r_{7m}h_{1m}^* - r_{8m}h_{2m}^* + r_{9m}^*h_{7m} + r_{11m}^*h_{5m} - r_{12m}^*h_{6m} - r_{13m}^*h_{3m} + r_{14m}^*h_{4m} - r_{15m}^*h_{1m} - r_{16m}^*h_{2m}$,

$\tilde{s}_8 = \sum_{m=1}^{N_r} -r_{2m}h_{7m}^* + r_{3m}h_{6m}^* + r_{4m}h_{5m}^* - r_{5m}h_{4m}^* - r_{6m}h_{3m}^* + r_{7m}h_{2m}^* - r_{8m}h_{1m}^* - r_{10m}^*h_{7m} + r_{11m}^*h_{6m} + r_{12m}^*h_{5m} - r_{13m}^*h_{4m} - r_{14m}^*h_{3m} + r_{15m}^*h_{2m} - r_{16m}^*h_{1m}$,

B.3: $N_t = 6$

$\tilde{s}_1 = \sum_{m=1}^{N_r} r_{1m}h_{1m}^* + r_{2m}h_{2m}^* + r_{3m}h_{3m}^* + r_{4m}h_{4m}^* + r_{5m}h_{5m}^* + r_{6m}h_{6m}^* + r_{9m}^*h_{1m} + r_{10m}^*h_{2m} + r_{11m}^*h_{3m} + r_{12m}^*h_{4m} + r_{13m}^*h_{5m} + r_{14m}^*h_{6m}$,

$\tilde{s}_2 = \sum_{m=1}^{N_r} r_{1m}h_{2m}^* - r_{2m}h_{1m}^* + r_{3m}h_{4m}^* - r_{4m}h_{3m}^* + r_{5m}h_{6m}^* - r_{6m}h_{5m}^* + r_{9m}^*h_{2m} - r_{10m}^*h_{1m} + r_{11m}^*h_{4m} - r_{12m}^*h_{3m} + r_{13m}^*h_{6m} - r_{14m}^*h_{5m}$,

$\tilde{s}_3 = \sum_{m=1}^{N_r} r_{1m}h_{3m}^* - r_{2m}h_{4m}^* - r_{3m}h_{1m}^* + r_{4m}h_{2m}^* - r_{7m}h_{5m}^* - r_{8m}h_{6m}^* + r_{9m}^*h_{3m} - r_{10m}^*h_{4m} + r_{11m}^*h_{1m} + r_{12m}^*h_{2m} - r_{15m}^*h_{5m} - r_{16m}^*h_{6m}$,

$\tilde{s}_4 = \sum_{m=1}^{N_r} r_{1m}h_{4m}^* + r_{2m}h_{3m}^* - r_{3m}h_{2m}^* - r_{4m}h_{1m}^* + r_{7m}h_{6m}^* - r_{8m}h_{5m}^* + r_{9m}^*h_{4m} + r_{10m}^*h_{3m} - r_{11m}^*h_{2m} - r_{12m}^*h_{1m} + r_{15m}^*h_{6m} - r_{16m}^*h_{5m}$,

$\tilde{s}_5 = \sum_{m=1}^{N_r} r_{1m}h_{5m}^* - r_{2m}h_{6m}^* - r_{5m}h_{1m}^* + r_{6m}h_{2m}^* + r_{7m}h_{3m}^* + r_{8m}h_{4m}^* + r_{9m}^*h_{5m} - r_{10m}^*h_{6m} - r_{13m}^*h_{1m} + r_{14m}^*h_{2m} + r_{15m}^*h_{3m} + r_{16m}^*h_{4m}$,

$\tilde{s}_6 = \sum_{m=1}^{N_r} r_{1m}h_{6m}^* + r_{2m}h_{5m}^* - r_{5m}h_{2m}^* - r_{6m}h_{1m}^* - r_{7m}h_{4m}^* + r_{8m}h_{3m}^* + r_{9m}^*h_{6m} + r_{10m}^*h_{5m} - r_{13m}^*h_{2m} - r_{14m}^*h_{1m} - r_{15m}^*h_{4m} + r_{16m}^*h_{3m}$,

$\tilde{s}_7 = \sum_{m=1}^{N_r} r_{3m}h_{5m}^* - r_{4m}h_{6m}^* - r_{5m}h_{3m}^* - r_{6m}h_{4m}^* - r_{7m}h_{1m}^* - r_{8m}h_{2m}^* + r_{11m}^*h_{5m} - r_{12m}^*h_{6m} - r_{13m}^*h_{3m} + r_{14m}^*h_{4m} - r_{15m}^*h_{1m} - r_{16m}^*h_{2m}$,

$\tilde{s}_8 = \sum_{m=1}^{N_r} r_{3m}h_{6m}^* + r_{4m}h_{5m}^* - r_{5m}h_{4m}^* - r_{6m}h_{3m}^* + r_{7m}h_{2m}^* - r_{8m}h_{1m}^* + r_{11m}^*h_{6m} + r_{12m}^*h_{5m} - r_{13m}^*h_{4m} - r_{14m}^*h_{3m} + r_{15m}^*h_{2m} - r_{16m}^*h_{1m}$, B.4: $N_t = 5$

$\tilde{s}_1 = \sum_{m=1}^{N_T} r_{1m}h_{1m}^{*} + r_{2m}h_{2m}^{*} + r_{3m}h_{3m}^{*} + r_{4m}h_{4m}^{*} + r_{5m}h_{5m}^{*} + r_{9m}^{*}h_{1m} + r_{10m}^{*}h_{2m} + r_{11m}^{*}h_{3m} + r_{12m}^{*}h_{4m} + r_{13m}^{*}h_{5m}$,

$\tilde{s}_2 = \sum_{m=1}^{N_T} r_{1m}h_{2m}^{*} - r_{2m}h_{1m}^{*} + r_{3m}h_{4m}^{*} - r_{4m}h_{3m}^{*} - r_{6m}h_{5m}^{*} + r_{9m}^{*}h_{2m} - r_{10m}^{*}h_{1m} + r_{11m}^{*}h_{4m} - r_{12m}^{*}h_{3m} - r_{14m}^{*}h_{5m}$,

$\tilde{s}_3 = \sum_{m=1}^{N_T} r_{1m}h_{3m}^{*} - r_{2m}h_{4m}^{*} - r_{3m}h_{1m}^{*} + r_{4m}h_{2m}^{*} - r_{7m}h_{5m}^{*} + r_{9m}^{*}h_{3m} - r_{10m}^{*}h_{4m} + r_{11m}^{*}h_{1m} + r_{12m}^{*}h_{2m} - r_{15m}^{*}h_{5m}$,

$\tilde{s}_4 = \sum_{m=1}^{N_T} r_{1m}h_{4m}^{*} + r_{2m}h_{3m}^{*} - r_{3m}h_{2m}^{*} - r_{4m}h_{1m}^{*} - r_{8m}h_{5m}^{*} + r_{9m}^{*}h_{4m} + r_{10m}^{*}h_{3m} - r_{11m}^{*}h_{2m} - r_{12m}^{*}h_{1m} - r_{16m}^{*}h_{5m}$,

$\tilde{s}_5 = \sum_{m=1}^{N_T} r_{1m}h_{5m}^{*} - r_{5m}h_{1m}^{*} + r_{6m}h_{2m}^{*} + r_{7m}h_{3m}^{*} + r_{8m}h_{4m}^{*} + r_{9m}^{*}h_{5m} - r_{13m}^{*}h_{1m} + r_{14m}^{*}h_{2m} + r_{15m}^{*}h_{3m} + r_{16m}^{*}h_{4m}$,

$\tilde{s}_6 = \sum_{m=1}^{N_T} r_{2m}h_{5m}^{*} - r_{5m}h_{2m}^{*} - r_{6m}h_{1m}^{*} - r_{7m}h_{4m}^{*} + r_{8m}h_{3m}^{*} + r_{10m}^{*}h_{5m} - r_{13m}^{*}h_{2m} - r_{14m}^{*}h_{1m} - r_{15m}^{*}h_{4m} + r_{16m}^{*}h_{3m}$,

$\tilde{s}_7 = \sum_{m=1}^{N_T} r_{3m}h_{5m}^{*} - r_{5m}h_{3m}^{*} - r_{6m}h_{1m}^{*} - r_{7m}h_{1m}^{*} - r_{8m}h_{2m}^{*} + r_{11m}^{*}h_{5m} - r_{13m}^{*}h_{3m} + r_{14m}^{*}h_{4m} - r_{15m}^{*}h_{1m} - r_{16m}^{*}h_{2m}$,

$\tilde{s}_8 = \sum_{m=1}^{N_T} r_{4m}h_{5m}^{*} - r_{5m}h_{4m}^{*} - r_{6m}h_{3m}^{*} + r_{7m}h_{2m}^{*} - r_{8m}h_{1m}^{*} + r_{12m}^{*}h_{5m} - r_{13m}^{*}h_{4m} - r_{14m}^{*}h_{3m} + r_{15m}^{*}h_{2m} - r_{16m}^{*}h_{1m}$,

B.5: $N_t = 4$

$\tilde{s}_1 = \sum_{m=1}^{N_T} r_{1m}h_{1m}^{*} + r_{2m}h_{2m}^{*} + r_{3m}h_{3m}^{*} + r_{4m}h_{4m}^{*} + r_{5m}^{*}h_{1m} + r_{6m}^{*}h_{2m} + r_{7m}^{*}h_{3m} + r_{8m}^{*}h_{4m}$,

$\tilde{s}_2 = \sum_{m=1}^{N_T} r_{1m}h_{2m}^{*} - r_{2m}h_{1m}^{*} - r_{3m}h_{4m}^{*} + r_{4m}h_{3m}^{*} + r_{5m}^{*}h_{2m} - r_{6m}^{*}h_{1m} - r_{7m}^{*}h_{4m} + r_{8m}^{*}h_{3m}$,

$\tilde{s}_3 = \sum_{m=1}^{N_T} r_{1m}h_{3m}^{*} + r_{2m}h_{4m}^{*} - r_{3m}h_{1m}^{*} - r_{4m}h_{2m}^{*} + r_{5m}^{*}h_{3m} + r_{6m}^{*}h_{4m} - r_{7m}^{*}h_{1m} + r_{8m}^{*}h_{2m}$,

$\tilde{s}_4 = \sum_{m=1}^{N_T} r_{1m}h_{4m}^{*} - r_{2m}h_{3m}^{*} + r_{3m}h_{2m}^{*} - r_{4m}h_{1m}^{*} + r_{5m}^{*}h_{4m} - r_{6m}^{*}h_{3m} + r_{7m}^{*}h_{2m} - r_{8m}^{*}h_{1m}$

B.6: $N_t = 3$

$\tilde{s}_1 = \sum_{m=1}^{N_T} r_{1m}h_{1m}^{*} + r_{2m}h_{2m}^{*} + r_{3m}h_{3m}^{*} + r_{5m}^{*}h_{1m} + r_{6m}^{*}h_{2m} + r_{7m}^{*}h_{3m}$,

$\tilde{s}_2 = \sum_{m=1}^{N_T} r_{1m}h_{2m}^{*} - r_{2m}h_{1m}^{*} + r_{4m}h_{3m}^{*} + r_{5m}^{*}h_{2m} - r_{6m}^{*}h_{1m} + r_{8m}^{*}h_{3m}$,

$\tilde{s}_3 = \sum_{m=1}^{N_T} r_{1m}h_{3m}^{*} - r_{3m}h_{1m}^{*} - r_{4m}h_{2m}^{*} + r_{5m}^{*}h_{3m} - r_{7m}^{*}h_{1m} + r_{8m}^{*}h_{2m}$,

$\tilde{s}_4 = \sum_{m=1}^{N_T} -r_{2m}h_{3m}^{*} + r_{3m}h_{2m}^{*} - r_{4m}h_{1m}^{*} - r_{6m}^{*}h_{3m} + r_{7m}^{*}h_{2m} - r_{8m}^{*}h_{1m}$

B.7: $N_t = 2$

$\tilde{s}_1 = \sum_{m=1}^{N_T} r_{1m}h_{1m}^{*} + r_{2m}^{*}h_{2m}$

$\tilde{s}_2 = \sum_{m=1}^{N_T} r_{1m}h_{2m}^{*} - r_{2m}^{*}h_{1m1}$