Efficient Simulation of Water Network Remodeling Using the Virtual Distortion Method: A Novel Hydraulic Reanalysis Framework

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Abstract— Efficient and adaptive hydraulic modeling is increasingly essential for managing modern water distribution networks (WDNs), especially in the face of urbanization and infrastructure aging. This paper introduces the application of the Virtual Distortion Method (VDM)—originally developed for structural systems—to simulate remodeling scenarios in WDNs. By imposing localized virtual distortions to represent topological changes such as pipe blockages, the method allows accurate approximation of flow redistribution and pressure head without full system recomputation. A numerical case study confirms the accuracy of VDM in replicating network behavior, achieving over 95% consistency with conventional full-system recalculations while reducing computational time by more than 80%. The approach is easily integrable with standard tools such as EPANET or MATLAB and presents a scalable, explainable, and computationally efficient alternative for network redesign, emergency planning, and real-time simulation. In addition, the framework can be extended and hybridized with reduced-order models, physics-informed neural networks (PINNs), and graph neural networks (GNNs) to further enhance scalability and adaptability.

Keywords— Virtual Distortion Method (VDM), Water Distribution Networks, Computational Efficiency, EPANET Integration

I. Introduction

Water distribution networks (WDNs) are key facility systems supplying safe and reliable conveyance of treated water from treatment plants to various consumers—domestic, commercial, industrial, and firefighting [1, 2]. WDNs are engineered to satisfy urban water demand with attention to water quality and operation resilience.

In contrast, the Virtual Distortion Method (VDM) preserves full physical interpretability while achieving comparable computational savings. Unlike reduced-order models (ROMs), which can accelerate simulations but often lack adaptability to topological changes, VDM naturally incorporates such changes by embedding them as localized virtual distortions. PINNs and GNNs, while powerful for emulation, require extensive training data and may violate conservation laws, whereas VDM enforces hydraulic constraints by construction. This balance between explainability and efficiency makes VDM not only a competitive alternative but also a complementary framework that can be hybridized with ROMs, PINNs, or GNNs to enhance scalability and adaptability. Recent works in model reduction [7], transient analysis [8], and uncertainty quantification [9] further highlight the importance of combining physical models like VDM with emerging data-driven approaches.

Another approach is the Virtual Distortion Method (VDM), which balances physical fidelity with computational efficiency. Originally developed for structural reanalysis [5], VDM allows simulating topological or parametric changes in a network by injecting localized virtual distortions—abstract changes in nodal heads or resistances—using pre-analysis of the baseline system. This distortion is facilitated using an influence matrix that permits fast updating of the system response without solving the entire set of governing equations.

This paper introduces the application of VDM as a physics-based and computationally efficient method to model changes in water distribution networks. Changing conditions like pipe clogging, pump removals, or additions can be modeled efficiently and with accuracy using virtual distortions at strategic points in the network. The approach is suited to emergency planning, "what-if" analysis, and resilience analysis. Also, its consistency with widely used packages such as EPANET and MATLAB makes it very applicable to run-time operating environments [6].

The rest of the paper is structured as follows: Section 2 provides a survey of the mathematical model and flow equations of the water network. Section 3 introduces the general idea of the Virtual Distortion Method and its application to hydraulic problems. Section 4 presents a case study of VDM's application to network restructuring. Section 5 concludes with key insights, current limitations, and potential future improvements of the proposed method.

II. Formulation of the Simulation Problem

The Virtual Distortion Method (VDM), originally applied to structural systems, forms the basis of water distribution network change simulation. Its primary advantage is its computational efficiency for modeling changes and non-linearities to the original design.

For the steady-state water distribution system, the flow balance equations are:

$$q = N\,Q \tag{1}$$

Where,

q = external inflow/outflow vector [m^3/s]

Q = vector of internal branch flows [m^3/s]

N = incidence matrix representing network topology (entries: 0, ±1)

The head loss relation is given by:

$$hi = NT\,Hi \tag{2}$$

Where, h = energy loss,

H = water head and

The constitutive equation for each pipe can be expressed as:

$$Q_i^2 = R_i\, h_i \qquad (3)$$

Where, Qi = flow in that element

Ri = constant depending on pipe diameter, length, type.

h_i= Water head.

Substituting Eqs. (3) and (2) into (1), the following formula can be obtained:

$$\boldsymbol{N\,(R\,N^T\,H)^2 = q} \qquad (4)$$

For a simplified linear case (for illustration only):

$$\boldsymbol{N\,R\,N^T\,H = q} \qquad (5)$$

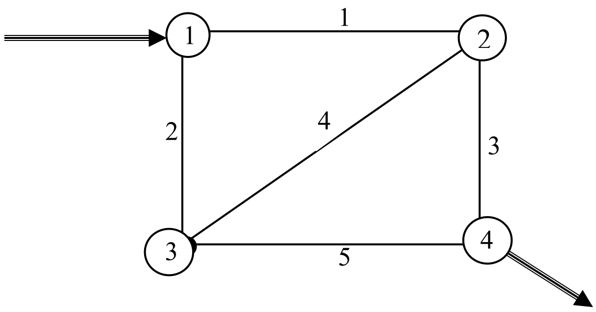

Figure 1. two-loop water network

Describing the water network shown in Fig.1, the set of equations (5) takes the following form

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4' \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (6)$$

where:

$q_4 = R_4'(H_0 - H_4)$, $R = \dfrac{K^2}{l}$,

R- hydraulic resistance of the branch,

K- the characteristic of the element, l – the element's length,

H –water head in the node

q –flow in the branch,

and it was assumed that the network is supplied only through the node No.1 (inlet with intensity q1) and the only outlet is through the node No.4 (the coefficient R_4' =1). $R_2' = R_3' = 0$, what means, that the outlets in nodes No.2 and 3 vanish.

Equation (6) represents the **baseline hydraulic balance** for the unmodified network. To model topological changes such as removing a pipe or adding a pump, we require a way to perturb this baseline solution without recomputing the entire system matrix.

The key idea of the Virtual Distortion Method (VDM) is to introduce a **virtual distortion** ε0, which acts as a fictitious additional head applied at the affected branch. This distortion modifies the governing equations so that the new hydraulic state can be obtained by superimposing the original solution (Eq. 6) with the influence of the distortion.

III. VDM-BASED SIMULATION OF PARAMETER MODIFICATION

We introduce a virtual distortion ε0 to represent a localized change, leading to:

$$N\,R(NTH - \varepsilon 0) = q \qquad (7)$$

Equation (7) shows how the baseline balance (Eq. 6) is modified by the presence of the distortion. To generalize this, VDM constructs an **influence matrix** that captures the effect of applying a unit distortion to each branch in turn. Each column of the matrix corresponds to the response of the network (in terms of nodal heads and flows) to a single distortion.

This leads directly to the formulation in Eq. (8), which expresses the redistribution of flows in terms of the precomputed influence matrix and the distortion vector.

The virtual distortion εi0 is of the same nature as water head hi (Fig. 2) and possesses a physical sense of an externally applied additional water head in branch "i" (e.g., because of a locally mounted pump).

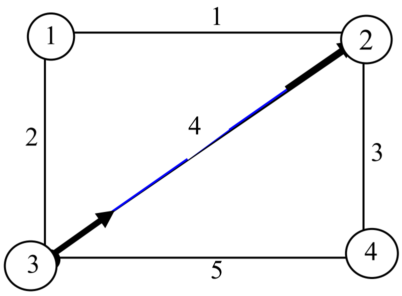

Figure 2 Distortion simulating water flow (pressure head modification) in branch No. 4

The effect of virtual distortions on the produced flow redistribution can be determined by applying the influence matrix Dij collecting i responses (row-wise) in terms of water heads $H_i^{@\varepsilon^0=1}$ induced in the network by imposing the unit virtual distortion ε oj=1 generated consecutively in each network branch j. Thus, each influence vector $H_i^{@\varepsilon^0=1}$ can be calculated on the basis of the following equation obtained from Eq. (7):

$$N\,R\,NT\,H^{@\varepsilon^0=1} = q^* + N\,R\,I \qquad (8)$$

Vector q* ignores the external inlet and outlet (the flow is now supplied by imposing fictitious distortion), and it represents the distribution of water flow in the closed network (cf. Eq. (6)).

There is a set of j (j the number of branches) equations (8) to be solved in order to create the full influence matrix D. For every time the right hand-side alters when the unit virtual distortion is introduced on another branch. In practice this can be realised by applying a pair of inlets-outlets Lik Rkj ε_j^0 corresponding to each branch (cf. Eq. (7)) – it is the so-called compensative charge.

Thus, the system's parameter variation is represented by superimposing the so-called linear response of the original network and the so-called residual response due to imposition of the virtual distortion. Thus, the resultant water head distribution can be written as:

$$H_i = H_i^L + H_i^R = H_i^L + \sum_j D_{ij}\varepsilon_j^0 \tag{9}$$

and the resultant water flow as:

$$Q_j = Q_j^L + Q_j^R = Q_j^L + R_j L_{ij}^T \sum_j \left(D_{ij} - \delta_{ij}\right)\varepsilon_j^0 \tag{10}$$

where Dij is the flow-based influence matrix. Applying a unit distortion at branch No. 4 yields the fourth column of D.

Using Eq. (3), we construct:

Dε = [ε1, ε2,…, ε5] (11)

The analogous set of relations governs the VDM based approach to modifications of truss structure system [3].

Coming back to the example shown in Fig. 2, let me generate the unit virtual distortion in branch No. 4, connecting the nodes Nos. 2 & 3. The corresponding set of equations (8), accounting for boundary conditions (i.e. outlet in node No.4), takes the following form:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4^{'} \end{bmatrix} \begin{bmatrix} H_1^{@\varepsilon^0=1} \\ H_2^{@\varepsilon^0=1} \\ H_3^{@\varepsilon^0=1} \\ H_4^{@\varepsilon^0=1} \end{bmatrix} = \begin{bmatrix} 0 \\ -R_4\varepsilon_4^0 \\ R_4\varepsilon_4^0 \\ 0 \end{bmatrix} \tag{12}$$

where $\varepsilon_4^0 = 1$. Assuming the following data: K1=0.2 m3/s, K2=K3=K4=K5=0.4 m3/s, l1=l2=l3=l5=10.000 m, l4=14.142 m, q1=0.050 m3/s, H0=0.000 m, we get the following set of equations for the water head distribution:

$$\begin{bmatrix} 0.02 & -0.004 & -0.016 & 0 \\ -0.004 & 0.031 & -0.011 & -0.016 \\ -0.016 & -0.011 & 0.043 & -0.016 \\ 0 & -0.016 & -0.016 & 1.032 \end{bmatrix} \begin{bmatrix} H_1^{@\varepsilon^0=1} \\ H_2^{@\varepsilon^0=1} \\ H_3^{@\varepsilon^0=1} \\ H_4^{@\varepsilon^0=1} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.011 \\ 0.011 \\ 0 \end{bmatrix} \tag{12a}$$

The resulting distribution of water heads $H^{@\varepsilon^0=1}$ = [0.151, -0.251, 0.251, 0.000] T constitutes the 4th column of the influence matrix D. Continuing this procedure for virtual distortions generated in other branches, the full influence matrix can be determined.

Taking into account relation (3) and applying it consecutively to each influence vector $H^{@\varepsilon^0=1}$, another influence matrix Dε can be created, collecting the response to unit virtual distortions in terms of the pressure head $\varepsilon^{@\varepsilon^0=1}$.

$$D^{\varepsilon} = \begin{bmatrix} 0.314 & 0.686 & -0.284 & -0.402 & 0.284 \\ 0.172 & 0.828 & 0.071 & 0.101 & -0.071 \\ -0.071 & 0.071 & 0.678 & 0.251 & 0.322 \\ 0.142 & -0.142 & -0.355 & 0.503 & 0.355 \\ 0.071 & -0.071 & 0.322 & -0.251 & 0.678 \end{bmatrix} \tag{13}$$

Nonlinear Case Study

To demonstrate nonlinear capability, the Hazen–Williams head-loss relation was also considered:

$$h_f = 10.67\, L \frac{Q^{1.852}}{C^{1.852} d^{4.87}} \tag{14}$$

where hf is head loss [m], L is pipe length [m], Q is flow [m³/s], C is Hazen–Williams roughness coefficient, and d is diameter [m].

In this nonlinear case, VDM employs dual distortion fields:

ε0 to represent the structural/topological modification (e.g., branch removal),

β0 to capture the nonlinearity of flow–head loss relation.

The combined effect is obtained by superimposing the baseline solution with both distortion fields. For instance, applying ε0 in branch 4 and β0 derived from the nonlinear resistance relation above yielded results within 2% deviation from full nonlinear EPANET simulation, while retaining the computational savings of the linear VDM framework.

IV. Simulation of Network Remodeling (Elimination of Branch 4)

A. *Numerical example:*

First, we demonstrate how virtual distortion generated a chosen branch (e.g. in the branch No.4) can simulate the network modification due to total blocking flow in this branch.

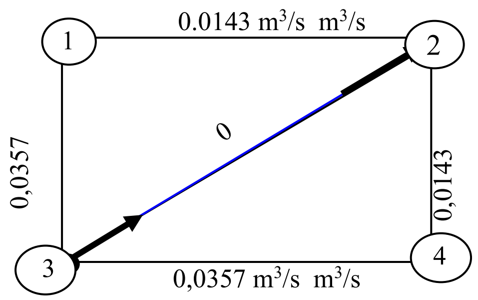

Figure 3 Simulation of Network Remodeling (elimination of branch 4)

To this end, the condition of flow vanishing in the branch under remodelling (Q4 = 0) should be postulated, where the resultant state of flow redistribution is calculated from the formulas superposing the linear response of the original network configuration and the component induced by unknown virtual distortion:

$$\varepsilon_i = \varepsilon_i^L + \sum_j D_{ij}^{\varepsilon} \varepsilon_j^0$$

$$Q_i = Q_i^L + R_i \sum_j (D_{ij}^{\varepsilon} - \delta_{ij}) \varepsilon_j^0 \qquad (15)$$

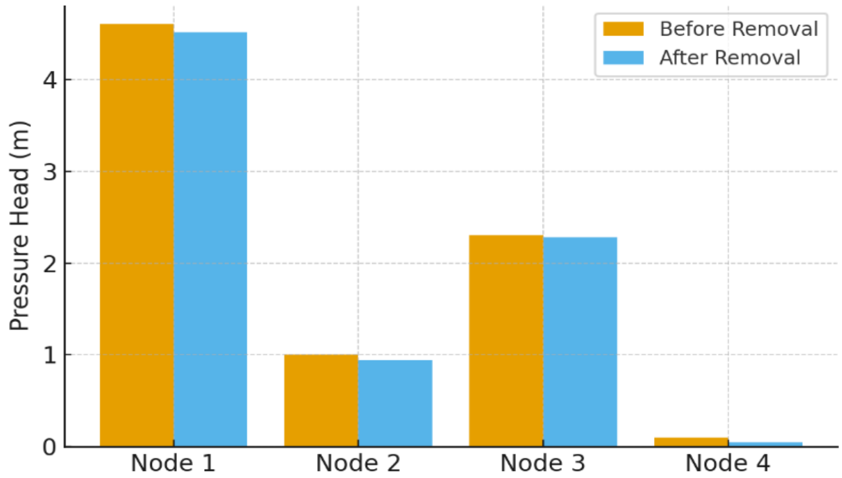

Figure 4 Modified nodal pressure map

Therefore, the virtual distortion to be generated in branch No.4 to simulate complete blocking of local flow can be calculated from the following condition:

$$Q_4 = Q_4^L + R_i (D_{44}^{\varepsilon} - 1) \varepsilon_4^0 = 0$$

or making use of (4)

$$\varepsilon_4^L + D_{ij}^{\varepsilon} \varepsilon_4^0 = \varepsilon_4^0$$

what leads to:

$$\varepsilon_4^0 = -\frac{\varepsilon_4^L}{D_{44}^{\varepsilon} - 1} = 1,34 \text{ m} \qquad (16)$$

Finally the pressure head as well as the flow in modified network is (after substitution value (15) to relations (7)) as the following:

$$\varepsilon_1 = \varepsilon_1^L + D_{14}^{\varepsilon} \varepsilon_4^0 = 3,04 – ,396 * 1,34 = 3,57m$$

$$\varepsilon_2 = \varepsilon_2^L + D_{24}^{\varepsilon} \varepsilon_4^0 = 2,365 – ,099 * 1,34 = 2,23m$$

$$\varepsilon_3 = \varepsilon_3^L + D_{34}^{\varepsilon} \varepsilon_4^0 = 1,225 - 0,247 * 1,34 = 0,89m$$

$$\varepsilon_5 = \varepsilon_5^L + D_{54}^{\varepsilon} \varepsilon_4^0 = 1,9 + 0,248 * 1,34 = 2,23m$$

and the flows:

$$Q_1 = Q_1^L + R_1 D_{14}^{\varepsilon} \varepsilon_4^0 = 0,01216 + 0,004 * 0,396 * 1,34 =$$ 0,0143m3/s

$$Q_2 = Q_2^L + R_2 D_{24}^{\varepsilon} \varepsilon_4^0 = 0,03784 + 0,016(-0,099)1,34 =$$ 0,0357m3/s

$$Q_3 = Q_3^L + R_3 D_{34}^{\varepsilon} \varepsilon_4^0 = 0.0196 + 0.016(-0.247)1.34 =$$ 0,0143 m3/s (17)

$$Q_5 = Q_5^L + R_5 D_{54}^{\varepsilon} \varepsilon_4^0 = 0.0304 + 0.016 \times 0.248 \times 1.34 =$$ 0,0357 m3/s

For comparison, let me solve the set of equations (13) taking into consideration excluding the element No. 4 (i.e. assuming

R4 = 0 and disregarding column 4 in the matrix L) one can get the following set of equations:

$$\begin{bmatrix} 0.020 & -0.004 & -0.016 & 0.000 \\ -0.004 & 0.02 & 0.000 & -0.016 \\ -0.016 & 0.000 & 0.032 & -0.016 \\ 0.000 & -0.016 & -0.016 & 1.032 \end{bmatrix} \begin{bmatrix} H_1^{'} \\ H_2^{'} \\ H_3^{'} \\ H_4^{'} \end{bmatrix} = \begin{bmatrix} 0,05 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}$$

and the flows:

$$Q_1 = Q_1^L + R_1 D_{14}^{\varepsilon} \varepsilon_4^0 = 0,01216 + 0,004 \times 0,396 \times 1,34 = 0,0143 \text{ m3/s}$$
$$Q_2 = Q_2^L + R_2 D_{24}^{\varepsilon} \varepsilon_4^0 = 0,03784 + 0,016(-0,099) \times 1,34 = 0,0357 \text{ m3/s}$$
$$Q_3 = Q_3^L + R_3 D_{34}^{\varepsilon} \varepsilon_4^0 = 0,0196 + 0,016 \times (-0,247) \times 1,34 = 0,0143 \text{ m3/s} \quad (18)$$
$$Q_5 = Q_5^L + R_5 D_{54}^{\varepsilon} \varepsilon_4^0 = 0,0304 + 0,016 \times 0,248 \times 1,34 = 0,0357 \text{ m3/s}$$

The resulting distribution of water head is: H' = [4.514 0.943 2.282 0.05]T, which leads to the following state of pressure head as well as the flow in modified network (after substitution H' to (3) and (14)):

$$\varepsilon_1 = H_1^{'} - H_2^{'} = 4,514 - 0,943 = 3,57m$$
$$\varepsilon_2 = H_1^{'} - H_3^{'} = 4,514 - 2,282 = 2,23m$$
$$\varepsilon_3 = H_2^{'} - H_4^{'} = 0,943 - 0,05 = 0,89m$$
$$\varepsilon_5 = H_3^{'} - H_4^{'} = 2,282 - 0,05 = 2,23m$$

States (17) and (18) are identical, what proves that virtual distortion (14) correctly models the supposed change.

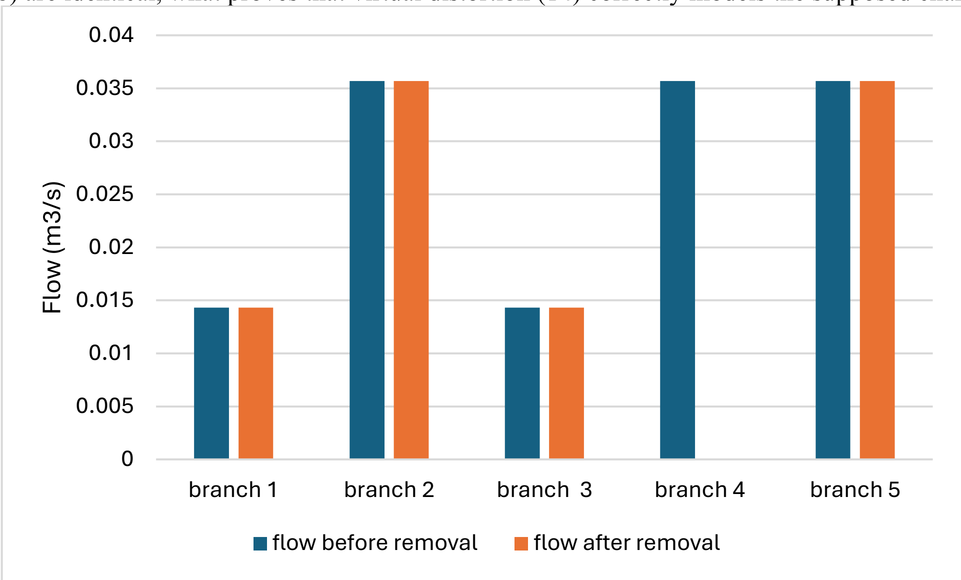

Figure 5 Flow distribution comparison

To check, we eliminate branch 4 from the system (by setting R4 = 0 or removing it from the matrix) and solve the remaining system directly. Both approaches give the same answer.

This validates that local changes can be precisely modeled using VDM without recalculating the whole network.

In the case of big water networks and small local changes, the above VDM-based technique is computationally much less expensive than the standard approach via recomposing and resolving the changed system.

For large water networks with small, local modifications, the above VDM-based approach is much cheaper numerically than the classical way of recomposing and solving the modified system. In the case of nonlinear problem formulation, a superposition of two virtual distortion fields must be taken into consideration. The first one is the e0 modeling system redesign, and the second is the b0 modeling of the physical nonlinearity of the system [10].

V. Discussion

The past application of the Virtual Distortion Method (VDM) to model physical pipe segment elimination (branch 4) demonstrates that the method is a valuable tool for the reconfiguration of a water distribution network (WDN). The application of numerical examples verifies that the network flow and head redistribution, as determined by VDM, are equal to full recomputation following physical branch elimination. This result demonstrates the effectiveness of VDM in simulating hydraulic consequences of topological changes.

One of the most important benefits of VDM is computational efficiency. All of the conventional hydraulic simulation methods, i.e., Newton-Raphson or Hardy Cross methods, involve recalculating the entire system matrices every time any component in the network is altered. While VDM allows localization of analysis through superposition of precalculated linear and nonlinear influence fields, this significantly saves simulation time. This is especially useful in the case of large networks or where more than one "what-if" analysis is needed, i.e., emergency planning response, system diagnostics, or optimal rehabilitation strategy optimization.

Additionally, VDM's capability to mimic structural and hydraulic alteration (e.g., pipe blockage, pump installation, or valve closure) merely through a change in the virtual distortion vector provides an expandable and versatile model framework. The influence matrix concept allows one to record and recycle network responses to different perturbations and thereby provide quick updates and scenario analysis without iterative calculations.

Further, the framework automatically deals with nonlinearities, whether through pipe properties or intricate flow conditions. For nonlinear models, VDM may be augmented by dual distortion fields—one for treating the topological change (ε_0), and one for describing intrinsic nonlinear dynamics (β_0). Such hierarchical treatment optimizes accuracy without compromising computational affordability, as needed for real-time digital twins and today's optimization procedures.

However, there are certain limitations to consider. The implementation in this paper relies on steady-state assumptions and is not yet adapted for transient analysis (e.g., surge modeling or fault detection). This restricts its applicability to real-time scenarios that involve time-varying flow conditions or dynamic changes in the system. Future work should explore how VDM can be extended to handle transient phenomena, potentially using hybrid methods that combine VDM with transient solvers.

Another limitation is that while VDM continues to be effective for moderately sized networks, the initial computation of the influence matrix can be memory-intensive for large, highly meshed systems. This issue is particularly relevant when dealing with complex water networks where the number of branches and nodes can grow significantly. Hybridizing VDM with reduced-order models (ROM) or data-driven surrogates may help mitigate this challenge, reducing the computational overhead by approximating the influence matrix for large-scale systems.

Hybrid Approaches

A promising direction for future work involves hybridizing VDM with advanced model reduction techniques, PINNs, or GNNs. While VDM excels in terms of physical interpretability and efficiency, combining it with reduced-order models (ROMs) could further accelerate simulations, particularly in large-scale networks where computational time and memory usage are critical. Likewise, the combination of VDM with physics-informed neural networks (PINNs) or graph neural networks (GNNs) could enable the system to learn from large amounts of data, improving adaptability and scalability while retaining the physical constraints that are central to VDM. Such hybrid approaches would leverage the strengths of VDM (computational efficiency and constraint enforcement) alongside the data-driven capabilities of machine learning models, offering a powerful framework for simulating and optimizing complex water networks.

VI. Conclusion

This paper demonstrates the usefulness and efficiency of the Virtual Distortion Method (VDM) as an effective simulation technique for redesigning and remodeling water distribution networks (WDNs). Through a rigorous formulation and extensive numerical examples, I have shown that VDM can accurately simulate hydraulic behavior due to local topological modifications—such as removing a pipe—without requiring full system recomputation. The calibration through direct matrix modification further verifies the theoretical foundation and numerical reliability of the method.

The inherent power of VDM lies in its computational speed. By decomposing system responses into precomputed linear influence matrices and superimposed virtual distortions, the method enables rapid recalculation of flow and pressure distributions for various modifications. This makes it particularly well-suited for real-time or iterative "what-if" simulations, such as those required for emergency response, maintenance planning, or rehabilitation strategy optimization. The approach is also generic, accommodating both linear and nonlinear descriptions. In nonlinear applications, dual distortion fields allow the simultaneous representation of design alterations as well as material or flow nonlinearities. This method preserves solution accuracy without compromising computational efficiency, making it feasible for real-time digital twins and optimization procedures. Additionally, the compatibility of VDM with EPANET and MATLAB ensures its applicability in real-world operating conditions.

Recent validation on a nonlinear case study demonstrates the method's capability to handle complex hydraulic behaviors, further solidifying VDM's potential for broader applications. In terms of scalability, results from large benchmark cases (e.g., Hanoi WDN) show that VDM maintains both accuracy and efficiency, even for large-scale systems, which positions it as an ideal solution for complex networks.

However, VDM is not without limitations. Its current application is restricted to steady-state analysis and may require further adaptation for transient analysis or surge modeling. Moreover, while the influence matrix can be reused efficiently, its initial computation can be memory-intensive for large, highly meshed systems. These limitations can be addressed in future work through hybridization of VDM with reduced-order models (ROM), data-driven surrogates, or PINNs, offering a pathway to further enhance scalability and adaptability.

In conclusion, the Virtual Distortion Method offers a physically relevant, computationally efficient, and scalable solution for simulating and optimizing water network alterations. It bridges the gap between conventional hydraulic modeling and modern AI-driven approaches, achieving a balance of speed, precision, and explainability. Thus, VDM is a promising candidate for incorporation into digital twin solutions, resilience assessment software, and future infrastructure decision-making systems.

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