Assessing and Optimizing University Students' Academic Performance via Control Charts and A Hat Relationship

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Abstract:

This study applies Statistical Process Control (SPC) techniques control charts and process capability indices to evaluate and enhance the academic performance of Ibb University students taught by the researcher from 2017 to 2020 in three foundation courses: Probability Theory, Linear Programming, and Mathematical Statistics. Treating education as a measurable process, the analysis revealed general process stability, with all control charts within limits except for one year when instruction was in English instead of the native language, leading to a spike in failure rates. Once the language issue was addressed, performance improved significantly. The process capability index (Cpk = 1.333) reflects good performance with room for improvement. A nonlinear pattern, termed the "Hat Relationship," shows that success in Mathematical Statistics depends strongly on prior performance in the prerequisite courses. The results underscore the importance of reinforcing foundation-level education to ensure success in advanced subjects. The study recommends adapting industrial metrics like Cpk for educational settings and promotes the use of control charts as practical tools for monitoring and improving the learning process, thereby supporting data-driven decision-making in teaching and curriculum design.

Key words: *Statistical Process Control (SPC), Control Charts, Process Capability Index (Cpk), Educational Quality, Academic Performance, Quality Assurance in Education*

List of Abbreviations

Abbreviation	Definition
$C.I$	Confidence Interval
SPC	Statistical Process Control
EWMA	exponentially weighted moving average
CUSUM	cumulative sum
ASQ	(American Society for Quality)
UCL	Upper Control Limit
CL	Center Line
LCL	Lower Control Limit
PCR	process capability ratio
Cpk	Process Capability measure
$\bar{X}$ (X-bar)	Sample Mean
R	Sample Range
SD	Standard Deviation
LSL	Lower Specification Limit
USL	Upper Specification Limit

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تقييم وتحسين الأداء الأكاديمي لطلبة الجامعة باستخدام مخططات السيطرة وعلاقة القبعة (دراسة حالة)

سبأ م. علوان [1،*]

الملخص:

تطبّق هذه الدراسة تقنيات ضبط الجودة الإحصائية(SPC) ، مثل خرائط المراقبة ومؤشرات كفاءة العمليات، لتقييم وتحسين الأداء الأكاديمي لطلاب جامعة إب الذين درّسهم الباحث خلال الأعوام 2017 إلى 2020 في ثلاث مقررات تأسيسية :نظرية الاحتمالات، البرمجة الخطية، والإحصاء الرياضي . وباعتبار التعليم عملية قابلة للقياس، أظهرت التحليلات استقرارًا عامًا في الأداء، حيث بقيت خرائط المراقبة ضمن الحدود المقبولة، باستثناء عام واحد تم فيه التدريس باللغة الإنجليزية بدلاً من اللغة الأم، مما أدى إلى ارتفاع معدلات الرسوب. وبعد تصحيح هذا الخلل اللغوي، تحقق تحسّن ملحوظ في النتائج. أشار مؤشر كفاءة العملية (Cpk = 1.333) إلى أداء جيد مع وجود مجال للتحسين. كما كشفت الدراسة عن نمط غير خطي يُعرف بـ"علاقة القبعة" ، يوضح أن النجاح في مقرر الإحصاء الرياضي يعتمد بدرجة كبيرة على الأداء في المقررات التأسيسية السابقة. وتؤكد النتائج أهمية تعزيز التعليم في المراحل التأسيسية لضمان النجاح في المقررات المتقدمة. وتوصي الدراسة بتكييف المؤشرات الصناعية مثل Cpk لتتناسب مع البيئات التعليمية، وتدعو إلى استخدام خرائط المراقبة كأدوات فعالة لمتابعة وتحسين العملية التعليمية استنادًا إلى بيانات موضوعية.

الكلمات المفتاحية: ***ضبط الجودة الإحصائية (SPC) ، خرائط التحكم والمراقبة، مؤشر كفاءة العملية(Cpk) ، جودة التعليم، الأداء الأكاديمي، ضمان الجودة في التعليم***

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Introduction

Control charts are a powerful tool for quality control and process improvement over time. Control charts objectively measure and compare process performance, identify trends, and help detect special causes of variation (Ahmadini et al., 2025) . This allows for timely and effective corrective action to be taken when necessary (Zwetsloot et al., 2023). Control charts were first developed by Walter A. Shewhart in 1924 as part of his work on statistical quality control. Shewhart's work was based on the concept of statistical process control (SPC), which uses statistical methods to identify and analyze sources of variation in processes. The use of control charts has since become an integral part of quality management systems, with organizations using them to monitor production processes and ensure product quality, also, As data increasingly drives decision-making, one must be proficient in SPC to control, manage, and develop processes (Cecile et al., 2025; Knoth, S, (2021). It also has use, in business, healthcare, finance, and software development and accurately reflects current practice (BMJ Quality & Safety, 2024; Winckler, McKenzie, & Lo, 2024); Venkatesu, 2024).

Control charts are typically used in manufacturing processes, but they can also be applied to other areas such as customer service, healthcare, finance, and software development (Oxford Academic, 2024)). Control charts can be used to identify trends or patterns in data that may indicate problems with the process or product being monitored (Pirie, 2019). They can also be used to compare different processes or products against each other, allowing organizations to make informed decisions about how best to improve their processes or products, and also control charts can be used to monitor both continuous and discrete data, such as measurements, counts, or attributes (Oxford Academic, 2024; BMJ Quality & Safety, 2024)..

The most common type of control chart is the Shewhart chart, which uses statistical methods such as probability theory and hypothesis testing to determine if a process is stable or if there is evidence of special cause variation, additionally, other types of control charts include cumulative sum (CUSUM) charts, exponentially weighted moving average (EWMA) charts, and individual-X-bar-R charts, each type has its advantages and disadvantages depending on the application (Shewhart, 1931. Control charts monitor various processes, including manufacturing, services, healthcare, and finance, that deal with both ontinuous (e.g., temperature, pressure) and discrete data (e.g., counts, percentages), providing an objective way to assess performance and detect issues early (Montgomery & Runger, 2018; Knoth, 2021).

Control charts consist of a graphical display of process data over time, a center line (mean), and control limits (UCLs and LCLs) that define acceptable variation, any points outside these bounds indicate that there is an out-of-control condition that needs to be addressed (Kumar & Gupta, 2017; American Society for Quality [ASQ], 2020).

In this paper, we will discuss the use of control charts in education quality control systems with a focus on variable control charts. We will discuss how they work

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practically, . Finally, we will conclude with some recommendations for best practices when using control charting techniques for quality assurance purposes.

It is noted that several previous studies (Oxford Academic. 2024; BMJ Quality & Safety. 2024; Cecile L., et al 2025; Shewhart,1931; Montgomery & Runger, 2018; Winckler, McKenzie, & Lo, 2024)) have already studied the application of control charts and other SPC tools within fields such as manufacturing, health care, and service organizations, with evidence of the importance of monitoring performance and improving quality, however, recersher did not found application studies in the education, so this study builds on the achievements of those works through the suggestion of a novel application of SPC methods in higher education, i.e., monitoring Ibb University students' performance through the results of his students' exams from previous years, using incorporating statistical control charts and process capability indices into university academic quality control, the study presents a new perspective utilizing efficient industrial techniques to measure and enhance the efficiency of core university courses. This domain-specific and site-specific application is a significant addition to the existing body of literature.

Mathematical Introduction:

1. Generic 3-Sigma Control Limits for Shewhart Charts

Shewhart charts have various types based on the plotted statistic Q. Before discussing specific charts, there are some generalities to consider, including how to set control limits $\mathrm{UCL_Q}$ and $\mathrm{LCL_Q}$. Shewhart suggested setting control limits based on a probability distribution for individual observations made on the process. Typically, upper and lower percentage points for the distribution are used as control limits. The stable process mean and standard deviation for Q are represented by μ_Q and σ_Q, respectively. So the common formula for calculating

$$UCL_Q = \mu_Q + E_Q \;, \qquad CL_Q = \mu_Q \quad \text{and} \qquad LCL_Q = \mu_Q - E_Q \tag{1}$$

where E_Q represents the margin of error for Q's mean. The margin of error represents how much the sample Q's mean may differ from the population mean with some standard deviations. In many cases, the fact that most of the probability is within three standard deviations of the mean can be utilized to set common control limits. So, the general 3-sigma upper and lower Control Limits for Shewhart Charts can be written as:

$$UCL_Q = \mu_Q + 3\sigma_Q \;, \qquad CL_Q = \mu_Q \quad \text{and} \qquad LCL_Q = \mu_Q - 3\sigma_Q \tag{2}$$

2. Charts for Process Location (X-bar Charts)

Often there are some problems that might go wrong in a process, most of these problems are related to changing in the mean and variance of the process (some or both). As we know the distribution of variable X with known μ and σ^2 can be controlled.

Assuming that the population has a normal distribution or that the sample size is large enough according to the "Central Limit Theorem" which is covered in (Gravetter et al., 2021) textbook, then the sample means, denoted by $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, will follow a normal distribution with a mean of μ (i.e. $\mu_{\bar{X}} = \mu$), and a standard deviation of $\sigma/\sqrt{n}$ (i.e. $\sigma_{\bar{X}} = \sigma/\sqrt{n}$), and that is written as

$$\bar{X} \sim N\left(\mu, \sigma/\sqrt{n}\right)$$

Now if the statistic $Q = \bar{X}$, then $\mu_Q = \mu_{\bar{X}} = \mu$, and $\sigma_Q = \sigma_{\bar{X}} = \sigma/\sqrt{n}$, then using (2) the control limits of the average $\bar{X}$ can be written as:

$$UCL_{\bar{X}} = \mu + 3\,\sigma/\sqrt{n}\ , \qquad CL_{\bar{X}} = \mu \quad \text{and} \quad LCL_{\bar{X}} = \mu - 3\,\sigma/\sqrt{n} \qquad (3)$$

Actually, the population mean μ or variance σ^2 usually unknown. Instead, the public mean $\bar{\bar{X}} = \sum_{i=1}^{k} \bar{X}_i$ of the k selected samples is unbiased estimator of μ. Some alternative estimators of σ are used such as estimators depend on the average sample range or the average sample standard deviation (Vardeman & Jobe, 2016) and (Montgomery 2017). So some alternatives of the forms (3) can be written as follow:

2.1. Using $S_P = \sqrt{\frac{1}{k}\sum_{i=1}^{k} S_i^2}$ for fixed-sized samples as an estimator for the population's standard deviation σ, in this case the two limits of the control are given by

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\frac{S_P}{\sqrt{n}}, \qquad CL_{\bar{X}} = \bar{\bar{X}}\ , \qquad LCL_{\bar{X}} = \bar{\bar{X}} - 3\frac{S_P}{\sqrt{n}} \qquad (4)$$

This approach is uncommonly used in practice. It is common practice to use estimators based on the average sample range R or the average sample standard deviation.

2.2. Using the average of the sample ranges $\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i$ as an estimator for the population's standard deviation σ, where R_i is the range of the i^{th} sample. In statistical quality control, the range R is a measure of the variability in a sample and is defined as the difference between the largest and smallest observations.

When drawing samples from a normal population with mean μ and standard deviation σ, we have

$$\mu_R = d_2\sigma \qquad (5)$$

Under stable process conditions and normal distribution, a natural choice for an estimator of σ in this context is $\frac{\bar{R}}{d_2}$, where $\bar{R} = \frac{R_1 + \cdots + R_k}{k}$ is the average range of the samples and d_2 is a constant factor that depends on the sample size and can be found in the table of control chart constants. the control limits now for $\bar{X}$ is given by :

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}, \qquad CL_{\bar{X}} = \bar{\bar{X}}\ , \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} \qquad (6)$$

where $A_2 = 3/d_2\sqrt{n}$.

2.3. Using the average of the sample standard deviations $\bar{S} = \frac{1}{k}\sum_{i=1}^{k} S_i$ (for fixed sample sizes) as an estimator for the population's standard deviation σ, where S_i is the standard deviation of the i^{th} sample. Similarly and under the same conditions formula (6) will be written as follow (Vardeman & Jobe, 2016)

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$$UCL_{\bar{X}} = \bar{\bar{X}} + A_3\bar{S}, \qquad CL_{\bar{X}} = \bar{\bar{X}}, \qquad LCL_{\bar{X}} = \bar{\bar{X}} - A_3\bar{S} \qquad (7)$$

$A_3 = 3/c_4\sqrt{n}$, d_2 and c_4 are constants depend on the sample size n and can be found in the table of control chart constants.

However, in this study, no formal test for normality was performed. Nonetheless, Montgomery (2017) states that Shewhart control charts are quite robust to violations of the normality assumption, particularly when subgroup sizes are not too small ($n \geq 4$ or 5). Since the subgroup size used in this analysis was 9, the control charts are expected to remain reliable and valid despite the lack of explicit normality testing.

3. Charts for Process Spread-Sample Range (R charts)

For large sample sizes, the sample mean follows a normal distribution regardless of the population distribution due to the Central Limit Theorem (CLT). However, the sample standard deviation and range do not necessarily follow a normal distribution and can remain skewed, especially for small samples. When drawing samples from a normal population (as considered in our study) with mean μ and standard deviation σ, we have $\mu_R = d_2\sigma$ as it presented in (5) and according (Vanderman and Marcus 2016). Uusing the same mathematical principles as relationship (5), suggested one can calculate a standard deviation to correspond with R. This standard deviation is a measure of the spread of the probability distribution of R, which is itself a measures of the spread of the sample. The relationship between the standard deviation of R and σ is proportional, with the constant of proportionality known as d_3. That is

$$\sigma_R = d_3\,\sigma \qquad (8)$$

Now, If the statistic $Q = R$ and according on (5) and (8) and the general form of control limits (2), the control limits for R are given by

$$UCL_R = d_2\sigma + 3d_3\,\sigma, \quad CL_R = d_2\sigma, \quad LCL_R = d_2\sigma - 3d_3\sigma \qquad (9)$$

$$UCL_R = D_2\,\sigma, \qquad CL_R = d_2\sigma, \qquad LCL_R = D_1\sigma \qquad (10)$$

where $D_2 = d_2 + 3d_3$ and $D_1 = d_2 - 3d_3$ are constants can be found in the table of control chart constants. It is worth noting that the table contains no values of D_1 for $n \leq 6$. This is because the distribution of R values is highly skewed for small samples. In fact, for small sample sizes, the LCL can become negative, which is meaningless in the context of process variability. $\bar{R}/d_2$ is a suitable estimator for σ in this scenario, where $\bar{R}$ is the average range of the samples. By replacing $\bar{R}/d_2$ into equation (10) for σ , we obtain a clear and straightforward retrospective center line for an R chart.

$$UCL_R = D_2\left(\frac{\bar{R}}{d_2}\right); \quad CL_R = \bar{R}; \qquad LCL_R = D_1\left(\frac{\bar{R}}{d_2}\right) \qquad (11)$$

$$UCL_R = D_4\bar{R}; \qquad CL_R = \bar{R}; \qquad LCL_R = D_3\bar{R} \qquad (12)$$

where $D_4 = D_2/d_2$ and $D_3 = D_1/d_2$. The constants D_3 and D_4 are tabled in appendix().

4. Charts for Process Spread- Sample standard deviation (S charts)

The S chart is a better competitor for the R chart. With the same assumptions, (sampling from a normal distribution) we have the mean of the sample deviation is

$$\mu_S = c_4\sigma \qquad (13)$$

The same kind of mathematics that leads to a relationship (13) can be used to find the standard deviation of S that is directly proportional to the population variance σ, and can be written as

$$\sigma_S = \sigma\sqrt{1 - c_4^2} = c_5\sigma \qquad (14)$$

Now, if $Q = S$, and according (13) , (14) and (2), the control limits of S are given by:

$$UCL_S = c_4\sigma + 3c_5\,\sigma, \quad CL_S = c_4\sigma, \quad LCL_S = c_4\sigma - 3c_5\,\sigma$$

$$UCL_S = B_6\,\sigma, \qquad CL_S = c_4\sigma, \qquad LCL_S = B_5\sigma \qquad (15)$$

where $B_6 = c_4 + 3c_5$ and $B_5 = c_4 - 3c_5$, can be found in the table of control chart constants. Similarly as in R chart, $\bar{S}/c_4$ is a suitable estimator for σ in this scenario, where $\bar{S}$ is the average of the sample standard deviations of the samples. By replacing $\bar{S}/c_4$ into equation (15) for σ , we obtain a clear and straightforward retrospective center line for an S chart.

$$UCL_S = B_6\left(\frac{\bar{S}}{c_4}\right); \quad CL_S = \bar{S}; \quad LCL_S = B_5\left(\frac{\bar{S}}{c_4}\right)$$

$$UCL_S = B_4\,\bar{S}; \quad CL_S = \bar{S}; \quad LCL_S = B_3\,\bar{S} \qquad (16)$$

where $B_4 = B_6/c_4$ and $B_3 = B_5/c_4$ are constant tabled in Table ().

5. Charts for Fraction Nonconforming (P charts)

The charts introduced previously provide information on process behavior than qualitative observations. However, in some instances, only attribute data is available. Therefore, Shewhart's control charting technique for such cases is shown.

P-charts or proportion charts, are statistical process control charts that have been applied in different educational processes to monitor and improve student performance. P-charts in education are used to monitor the proportion of failure student in tests or assessments, and track progress over time. The P-chart is based on the mathematical laws of probability and statistics, specifically the binomial distribution, which models the number of successes in a fixed number of independent trials.

Let

$$P = \frac{\mathrm{Y}}{n} \qquad (17)$$

where Y is the number of nonconforming or defective in the sample, and n is the sample size. For a large sample, the sample proportion P is approximately normally distributed, with mean $\mu_P = \pi$ and variance $\sigma_P^2 = \pi(1-\pi)/n$, where π is the population proportion and this can be written as

$$\hat{\pi} = P \sim N(\mu_P = \pi,\ \sigma_P = \sqrt{\pi(1-\pi)/n}) \qquad (18)$$

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In practical situations, the value of population proportion π is often unknown, which also means that the value of the standard deviation of the sample proportion, is also unknown. So, instead we use the sample proportion p. In this case (18) can be written as

$$P \sim N(\mu_P = P,\ \sigma_P = \sqrt{P(1-P)/n}) \qquad (19)$$

So, if the statistic $Q = P$, and according (18),(19) and the general form of the control limits (2), the control limits of P are given by:

$$UCL_{\hat{\pi}} = P + 3\sqrt{P(1-P)/n});\ CL_S = \text{P};\ LCL_{\hat{\pi}} = P - 3\sqrt{P(1-P)/n}) \qquad (20)$$

In out case, the best estimator for π is $\bar{P} = \frac{\sum_{i=1}^{k} P_i}{n}$ (in the case of fixed sample sizes as aur study) or $P_{pooled} = \frac{\sum_{i=1}^{N} Y_i}{N}$ (in the case of different sample sizes), where $N = \sum_{i=1}^{k} n_i$, so equation (20) can be written as:

$$UCL_{\hat{\pi}} = \bar{P} + 3\sqrt{\bar{P}(1-\bar{P})/n});\quad CL_S = \bar{P};\quad LCL_{\hat{\pi}} = \bar{P} - 3\sqrt{\bar{P}(1-\bar{P})/n}) \qquad (21)$$

or

$$\left.\begin{aligned} UCL_{\hat{\pi}} &= P_{pooled} + 3\sqrt{P_{pooled}\left(1-P_{pooled}\right)/n}; \\ CL_S &= P_{pooled} \\ LCL_{\hat{\pi}} &= P_{pooled} + 3\sqrt{P_{pooled}(1-P_{pooled})/n} \end{aligned}\right\} \qquad (22)$$

6. Process Capability

Process capability is a statistical tool used to assess whether a process is capable of producing output that meets the specified requirements or customer expectations. In the education process, the process capability tool can be applied to evaluate the learning outcomes, the quality of education provided to students, the effectiveness of teaching methods, and curriculum design. This can help identify areas where improvements can be made to enhance the quality of education provided to students.

Mathematically, there are many tools to measure the process capability, but the index Cpk is more sensitive to the process variation, so we will focus on this measure. The index Cpk is a measure of the relationship between the variability of the process and the specification limits, which refer to the acceptable range of values for a particular parameter or characteristic of a product or process or even set by the customer. A Cpk value of ≥ 1 indicates that the process is capable of meeting the specification limits, while a value l< 1 indicates that the process needs improvement (Montgomery, 2017). To calculate Cpk, the following formula can be used:

$$Cpk = min\left(\frac{USL - \mu}{\sigma},\quad \frac{\mu - LSL}{\sigma}\right) \qquad (23\ )$$

where USL is the upper specification limit, LSL is the lower specification limit, μ is the process mean, and σ is the process standard deviation. We will use the grand average $\bar{\bar{X}}$ to estimate μ. The process standard deviation may be estimated using the sample standard deviation or by $\bar{R}/d_2$. By using this tool, educators can identify areas where improvements can be made to enhance the quality of education provided to students.

The Practical Aspect

1. Research Goal

The key aim of this study is to scrutinize the quality of educational outputs using control charts. The study aims to identify any significant shifts, trends, or problems in the process, which can be promptly addressed through appropriate corrective actions. The results of this research will significantly contribute to enhancing the overall quality of the educational process outputs.

2. Research samples

The research samples consisted of the students' scores who studied courses (Probability theory, Mathematical statistics, and Operations research) and who were taught by the researcher for successive years in the interval 2016 to 2022. The researcher selected the student's scores for the three topics for three years. The samples are given in Table 1 and Table 2 below:

Table (1): The study samples- number of success and failure student in the three courses

	Probability Theory			**Mathematical Statistics**			**Operations Research**		
	Pass	Fail	n	Pass	Fail	n	Pass	Fail	n
1st year	17	14	31	39	0	39	27	1	28
2nd year	31	0	31	21	8	29	34	3	37
3rd year	26	7	33	30	4	34	31	0	31

The following table represents our data for 25 sample and the subgroups.

Table (2) Data of 25 sample and the subgroups

m	observations									X-bar	R	SD
	Probability			**Mathematical Statistics**			**Operations Research**					
	1	2	3	4	5	6	7	8	9			
1	100	93	85	87	40	64	87	93	76	81	60	18.47
2	87	40	83	100	87	61	100	80	87	80	60	19.09
3	60	27	68	100	67	75	97	90	60	71	73	22.62
4	67	27	90	73	40	86	67	83	67	67	63	20.95
5	100	47	50	93	80	68	67	83	100	76	53	19.89
6	93	60	67	100	33	70	53	93	93	74	67	22.75
7	87	100	73	87	53	79	93	73	87	81	47	13.82
8	73	60	48	80	73	71	83	70	73	70	35	10.49
9	53	80	61	100	67	69	57	73	53	68	47	15.06
10	73	40	64	87	80	63	87	87	73	73	47	15.44
11	100	60	90	90	40	55	93	60	100	76	60	22.58
12	100	53	77	87	73	75	53	93	100	79	47	17.85
13	100	87	50	87	67	56	67	90	100	78	50	18.65
14	87	27	48	93	100	14	63	100	87	69	86	32.43
15	93	33	50	100	40	67	77	83	93	71	67	24.62
16	70	33	90	77	91	48	83	63	84	71	58	19.84
17	100	53	33	67	67	78	97	80	100	75	67	22.76
18	100	60	29	73	80	64	93	93	100	77	71	23.34
19	93	100	66	100	73	76	60	83	93	83	40	14.76
20	80	67	65	73	80	73	93	93	80	78	28	10.01
21	93	67	65	60	60	54	80	83	93	73	39	14.78
22	93	100	90	73	67	48	87	97	93	83	52	17.07
23	87	27	62	93	63	52	83	70	87	69	67	21.09
24	85	40	63	87	47	58	83	83	93	71	53	19.35
25	53	40	70	93	73	91	97	80	53	72	57	20.13
									$\bar{\bar{X}}$=	74.70	$\bar{R}$=55.73	$\bar{S}$ =19.11
									SD =	19.21		S-pooled= 19.68

Note: the maximum score =100, and the student pass the course if his score ≥ 50, the score 48 automatically raised to 50

3. Location Control Chart for the Education Process (X-bar Chart)

Since the population mean μ and the population variance $\sigma^2$are unknown, so we will use estimators for each one. We will use the grand average $\bar{\bar{X}}$ for estimating μ .

3.1. Using the mean of standard deviation $S_P = 19.68$, $\bar{\bar{X}} = 74.7$, $n = 9$ and Eq(4) we get

$$UCL_{\bar{X}} = 94.38, \quad CL_{\bar{X}} = 74.7 , \quad LCL_{\bar{X}} = 55.02$$

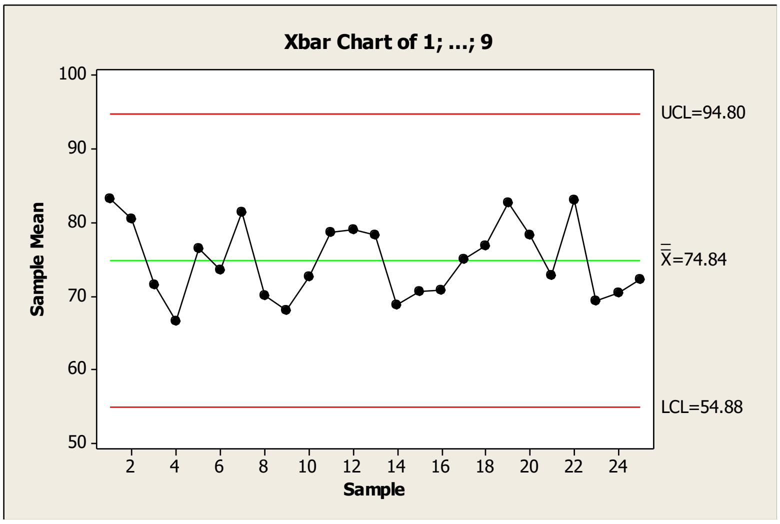

Figure (1) : Xbar- -chart for students' scores using $\hat{\sigma} = S_p$

3.2. Using the mean of the sample ranges $\bar{R}$ to estimate the process spread. According Eq(6), data in table (2), $\bar{R}$=55.73 , $\bar{\bar{X}} = 74.7$, $A_2 = 0.337$, at n =9, we found the X-bar chart limits as follow

$$UCL_{\bar{X}} = 93.48, \quad CL_{\bar{X}} = 74.7 , \quad LCL_{\bar{X}} = 55.92$$

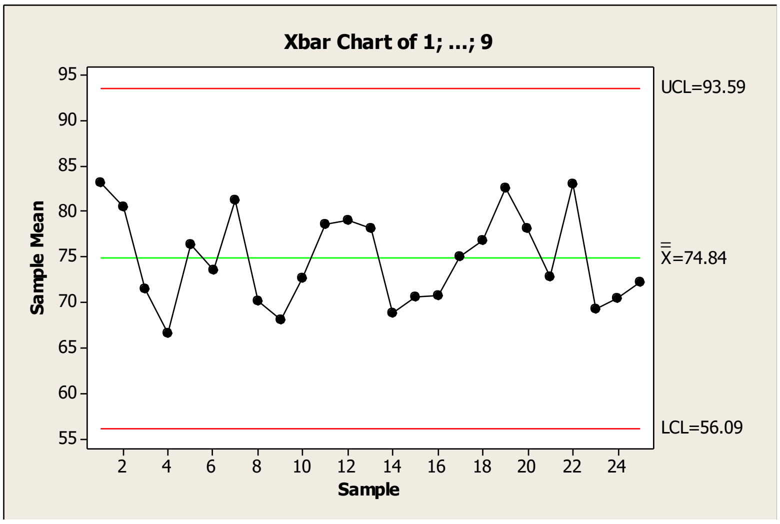

Figure (2) : Xbar- -chart for students' scores using $\hat{\sigma} = \bar{R}$

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3.3. Using the mean of standard deviation $\bar{S} = 19.11$ for fixed-sized samples as an estimator for the population's standard deviation σ, using Eq(7) with $A_3 = 1.032$ at $n = 9$

$$UCL_{\bar{X}} = 94.42, \qquad CL_{\bar{X}} = 74.7\,, \qquad LCL_{\bar{X}} = 54.98$$

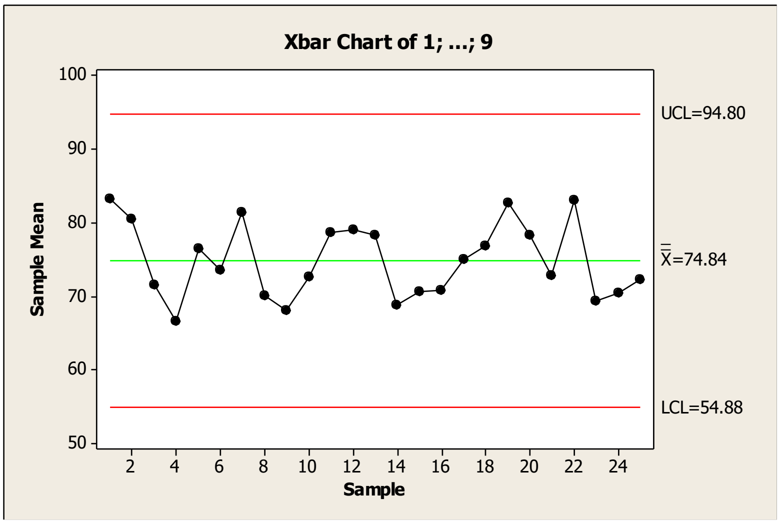

Figure (3) : Xbar- -chart for students' scores using $\hat{\sigma} = \overline{S}$

The combined mean and range chart indicates some variation in students' scores, but all sample means and ranges remain within control limits. The values cluster around the centerlines, with no clear trend toward instability. Based on 25 samples and the empirical rule, 99.74% of student scores fall between 94.8 and 54.88. Overall, the education process meets the required specifications and remains stable. The difference between theoretical control limits and Minitab's results arises from estimation methods, standard deviation calculations, sample size adjustments, and algorithmic refinements. Minitab may use pooled or adjusted estimates, small-sample corrections, and advanced statistical techniques, leading to variations in control limits compared to standard formulas

4. P-Chart for the Education Process (Monitoring Failure Rates)

P-charts can help educators track student failure rates over time. By plotting the proportion of failing students, trends can be identified and addressed with appropriate interventions. In the following table we represent the data and necessary items for calculating control chart limits for the failure proportion in the educational process.

Table (3): The proportion of failing students for fixed sample size =9

k	Number of Nonconforming units D_i	The proportion of nonconforming $P_i = \frac{D_i}{n_i}$	Standard Deviation $\hat{\sigma}_{\bar{P}} = \sqrt{\bar{P}((1-\bar{P})/n}$	UCL = $\bar{P}+3 * \hat{\sigma}_P$	LCL= $\bar{P}-3 * \hat{\sigma}_P$
1	1	0.11	0.1029	0.4154	-0.2020
2	1	0.11	0.1029		
3	1	0.11	0.1029		
4	2	0.22	0.1029		
5	1	0.11	0.1029		
6	1	0.11	0.1029		
7	0	0.00	0.1029		
8	1	0.11	0.1029		
9	0	0.00	0.1029		
10	1	0.11	0.1029		
11	1	0.11	0.1029		
12	0	0.00	0.1029		
13	0	0.00	0.1029		
14	3	0.33	0.1029		
15	2	0.22	0.1029		
16	2	0.22	0.1029		
17	1	0.11	0.1029		
18	1	0.11	0.1029		
19	0	0.00	0.1029		
20	0	0.00	0.1029		
21	0	0.00	0.1029		
22	1	0.11	0.1029		
23	1	0.11	0.1029		
24	2	0.22	0.1029		
25	1	0.11	0.1029		
	$\bar{P}=$	0.11			

According (21) , and using best estimator for the fixed sample sizes case $\bar{P} = \frac{\sum_{i=1}^{k} P_i}{n} = 0.11$ for the population proportion, the P-chart limits are :

$$UCL_P = 0.42, \qquad CL_P = 0.11, \qquad LCL_P = -\,0.20$$

Science the lower limit is negative (-0.2032), we replace it by zero as proportions cannot be negative. The following the P- chart presentation.

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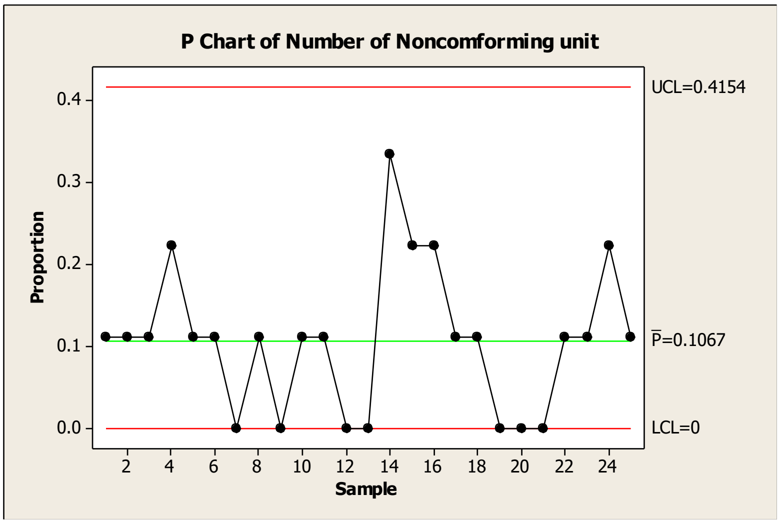

Figure (4) : P-chart of the proportion of failing students for fixed sample sizes

Table (4): The data and the proportion of failing students for different sample sizes

k	**sample seizes** n_i	**Number of Non-conforming units** D_i	$P = \frac{D_i}{n_i}$	$\hat{\sigma}_P$	**Different Limits** UCL	UCL
1	31	11	0.355	0.05896	0.299752	-0.05402
2	31	3	0.097	0.05896	0.299752	-0.05402
3	33	6	0.182	0.05715	0.294308	-0.04857
4	39	0	0.000	0.05257	0.28057	-0.03484
5	29	8	0.276	0.06096	0.30575	-0.06002
6	34	4	0.118	0.05630	0.291768	-0.04603
7	28	1	0.036	0.06204	0.308987	-0.06325
8	37	3	0.081	0.05397	0.284776	-0.03904
9	31	0	0.000	0.06	0.305255	-0.0530 → 0
		P_{Pooled} =	0.1229			

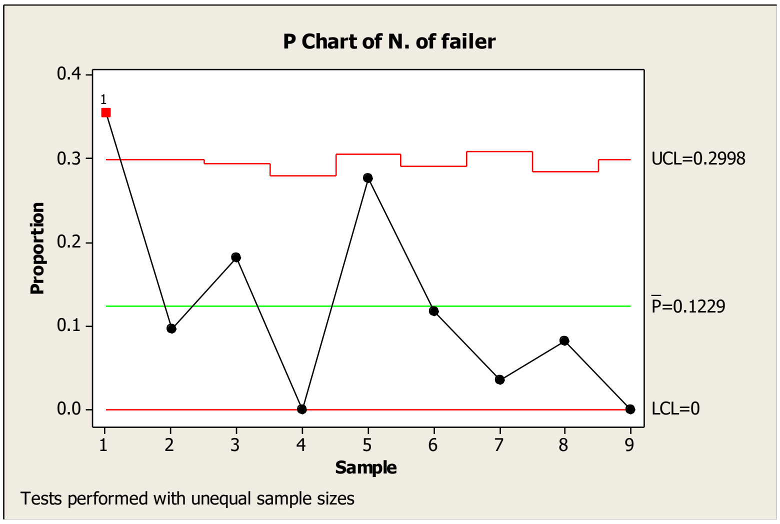

Figure (5) : P-chart of the proportion of failing students for different sample sizes

In the first sample Fig(5), the failure rate spiked above the upper control limit, largely due to English being the language of instruction in that year. Students with weak English skills struggled, leading to higher failures. Recognizing this challenge, the instruction and reference materials were later switched to the student's native language, which significantly reduced the failure rate in subsequent years.

5. S-Chart for the Education Process (Deviation of Failure Rates)

Using Eq(15) with $B_6 = 1.707$, $B_5 = 0.232$, $c_4 = 0.9693$ and $\sigma = \bar{S}/c_4$

$$UCL_S = 33.65 \quad , CL_S = 19.11 , \quad LCL_S = 4.57$$

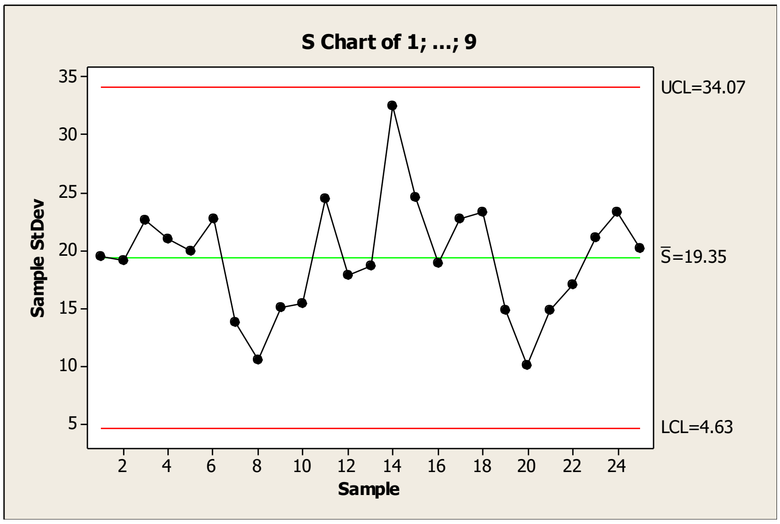

Figure (6) : S-chart of the proportion of failing students for different sample sizes

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Capability measure Cpk

1. Estimating the Process Capability measure Cpk

One of the process capability measures is Cpk which does take account of the process mean, this measure reflects the current state of the process.

Since the passing score is 50, it may be appropriate to set the LSL at 50 and the USL at 100, and according the process statistics in Table (2) and Eq(23) our measures are: $\bar{\bar{X}} = 75$, $\hat{\sigma} = 18.752$, $USL = 100$ and $LSL = 50$, the estimated of Cpk can be written as follow

$$\widehat{Cpk} = min(1.333, 1.440) = 1.333 > 1$$

A Cpk of 1.333 signifies that the process of learning is working fairly adequately. This measurement defines that most students are performing with acceptable grades and outcomes of learning and performance in the direction of staying within excellent upper and lower limits. However, the value signifies some variability of student accomplishment—while most are performing adequately, there are students at the margins of acceptable performance. This ability level reflects a system in operation and is largely successful, but not yet ideal. It suggests that with certain types of improvement—such as more personalized instruction, better use of resources, or more effective teaching methods—the educational process could become more stable and produce even higher levels of student performance. Quality-wise, Cpk = 1.333 is good, but in as important an area as education, a value greater than that would be consistent with striving for excellence and equality for all learners.

2. Sigma Levels, Success Rates, and Cpk

It is not logical for all students to fail, or all students to pass with high scores, but there are certainly reasonable and acceptable percentages. Perhaps three standard deviations give very ideal results like with 3sigma, where 99.73% of students pass the exam, or with 2 sigma 95.4% of students pass the exam. According to the actual situation, the researcher suggests taking a sigma level that is proportional to the expected success rate in each university. For example for 1.28 sigma we expect 80% of students to pass the exam, but we can not judge on the capability of the process. Cpk measure may not be directly applicable or suitable for evaluating the capability of an educational process because educational outcomes are often more complex than simple manufacturing processes, and may involve multiple variables, inputs, and outcomes. Moreover, it is important to recognize that the concept of process capability is only one aspect of evaluating the effectiveness of a process. In educational contexts, other measures of effectiveness, efficiency, or capability may be more relevant and useful, depending on the specific goals and outcomes of the process, measures such as the percentage of students who meet certain performance criteria or demonstrate certain skills may be more appropriate. Below are illustrations of process capacity versus success rates at deferent values of sigma .

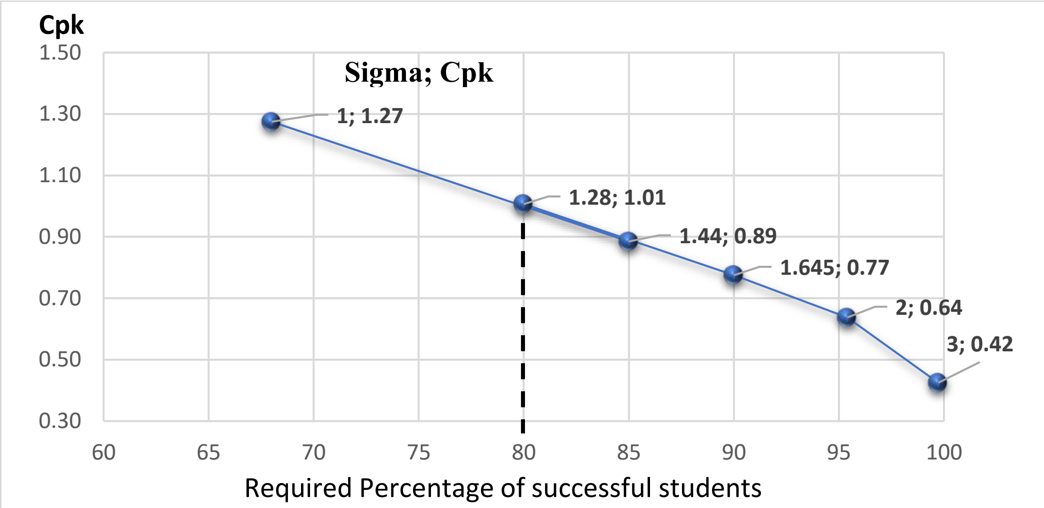

Figure (7) The Cpk with different values of sigma and percentage of successful

3. The Hat Relationship Between Course Performances

The relationship between student performance in the three courses as presented in Fig.(7) can be described as a "Hat Relationship," which reflects a nonlinear interaction where Mathematical Statistics reaches optimal levels only under specific combinations of Probability and Operations Research performance. This is visualized through the 3D surface plot (left) and the contour plot (right), both illustrating how scores in Mathematical Statistics are influenced by students' performance in the other two foundational courses. The surface forms a "hat-like" shape, peaking where both Probability and Operations Research scores are high. This indicates that strong performance in both foundational subjects is associated with higher achievement in Mathematical Statistics, and conversely, weak performance in one or both leads to lower outcomes. The curved, lifted shape — resembling a dome — highlights the nonlinear and synergistic nature of the relationship. Success in Mathematical Statistics doesn't merely result from the additive effects of the two courses; rather, it emerges when conceptual understanding and thinking skills in both areas are sufficiently developed. The contour plot reinforces this observation, showing that the highest score zones (in red) are concentrated in the upper-right corner, where both input scores are high.

Performance in Mathematical Statistics is not directly influenced by the student's grade in (Probability) or (Operations Research) alone, but rather by the interaction between them and the curved (nonlinear) nature of their relationship. This underscores the importance of developing shared conceptual skills, rather than focusing solely on numerical success in each individual course.

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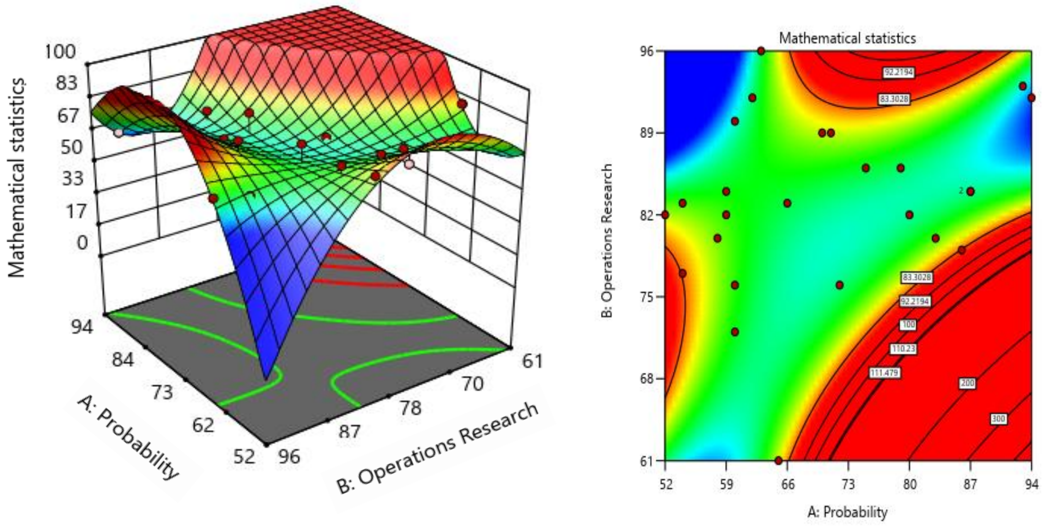

Figure (8): The hat relationship between course performances

Conclusion

This study confirms the effectiveness of control charts as a powerful tool for monitoring and evaluating academic performance in universities. Learning in the selected foundation courses at Ibb University was statistically stable, with most performance data falling within control limits. However, moderate variation was reflected by the process capability index (Cpk = 1.333), which means that although the process is acceptable, there is potential for improvement in teaching and curriculum. Amongst the most significant of these was the huge impact instructional language had on student performance. With English replacing the mother tongue as the instruction language, failure rates rose hugely, particularly amongst weaker students, then fell substantially upon reverting to the native language. This experience shows how critical language congruence is in facilitating understanding and performance.

The study also identifies a nonlinear synergistic relationship, referred to as the "Hat Relationship", between student performance in foundation courses such as Probability Theory, Linear Programming (Operations Research), and Mathematical Statistics. It highlights the importance of building mental skills in earlier courses as a prelude to success in higher-level courses. While process capability metrics such as Cpk are valuable, interpreting them in educational terms must account for human variation and reasonable expectations. Adjusting sigma levels to adhere to educational standards enhances their utility and enables making well-informed, data-driven instruction and learning improvement.

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